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Analysis and evaluation of standardized tests in arithmetic

BOSTON UNIVERSITY
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THESIS:

ANALYSIS and EVALUATION of STANDARDIZED TESTS in ARITHMETIC

Submitted by:
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Chapter I

The Problem

The purpose of this study is to analyze and evaluate representative standardized tests in arithmetic, and to so arrange and interpret the data that it may be found helpful in choosing or rejecting a standardized test.

Tests have accumulated rapidly. Hildredth's [1] 1933 summary, lists one hundred and forty five titles under Arithmetic Tests. This list covers not only the present available tests but others now out of print. Some arithmetic tests have been constructed for specific purposes but were later published for general use. Some who have constructed tests have been chiefly guided by statistical considerations, forgetting the criteria properly dictated by the purposes and uses of arithmetic. Many have used tests with little or no regard for purposes.

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1. Hildredth, Gertrude H., Bibliography Mental Tests and Rating Scales, Psychological Corporation, New York, 1933

Thus we arrive at a stage in the development of arithmetic tests when critical analysis and evaluation are needed. This will involve careful analysis of the content of each test studied, the setting up of criteria, and the application of the criteria in a fair but critical manner to the content, method, and general import of the tests. This is the task set for the present study.

McMurry[1], as long ago as 1904, called attention to the fact that many processes taught in arithmetic had little or no social value. The theory of social utility sets up the criterion of social usefulness as the chief basis for the selection of subject matter for a given course. Arithmetic is a tool subject. Assuming that arithmetic is justified in the grades on the basis of its utility in the common affairs of life, three phases of it need attention. 1. Experience in life situations in which there is a felt need for the knowledge of arithmetic; thus drill becomes meaningful. 2. Mastery of the useful number facts and the four fundamental processes. 3. Training in everyday life and business situations.

"The social point of view is sometimes characterized as being utilitarian. It may be so; but not in any narrow or undesirable sense. It

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1. McMurray, Frank, Proceedings of the National Education Association, July 1927, pages 194 - 202

demands that training be as wide as life itself. The content of courses in mathematics is to be determined by human needs. A fundamental need of our scientific age is more accurate quantitative thinking about our vocations, civic problems, taxation, income, insurance, expenditures, public improvements, and other public and private problems involving quantities. Arithmetic teaching like the teaching of penmanship is for the purpose of providing tools to be used in matters that lie beyond. Knowledge and drill should be given in their relation to the human activities in which they are used."[1]

Dewey[2] in The School and Society says, "There should be a natural connection of the everyday life of the child with the business environment

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1. Bobbitt, Franklin, What the Schools Teach and Might Teach, The Survey Committee of the Cleveland Foundation, Cleveland, Ohio, 1915

2. Dewey, John, The School and Society The University of Chicago Press, Chicago, Illinois, Revised Edition, Chapter III, page 66

about him, and it is the affair of the school to bring it into consciousness by keeping alive the ordinary bonds of relation. The subject of compound business partnership is probably not in many arithmetics nowadays., though it was there not a generation ago, for the makers of textbooks said if they left out anything they could not sell the books. This compound-business partnership originated as far back as the sixteenth century. The joint stock company was invented; compound partnership disappeared but the problems relating to it stayed in the arithmetics as useless material for two hundred years. They were kept after they ceased to have practical utility, for the sake of mental discipline. A great deal of what is now in the arithmetic under the head of percentage is of the same nature. Children of twelve and thirteen years of age go through gain and loss calculations, and various forms of bank discount so complicated that bankers dispensed with them long ago. The child should study his arithmetic not as an isolated thing by itself, but in reference to his social environment.

The youth needs to be acquainted with the bank as a factor in modern life, with what it does, and how it does it, and then relevant arithmetic processes would have some meaning - quite in contradiction to the time - absorbing, and mind - killing examples in percentage found in our textbooks.

"The school should be connected with life so that the experience gained by the child in a familiar, commonplace way is carried over and made use of there, and what the child learns in school is carried back and applied in everyday life."

Investigations have been made to determine as carefully as possible what arithmetic is used in the daily activities of everyday life. Dr. G. M. Wilson[1] conducted a survey of the social and business usage of arithmetic. The study was based on 14,583 problems contributed by 4,068 different persons representing 155 different occupations. Eighty five percent of all the

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1. Wilson, Guy M., A Survey of the Social and Business Usage of Arithmetic, Contributions to Education, Number 100, Teachers' College, Columbia University

problems involved the use of money in either buying or selling of goods. Multiplication, addition, subtraction, division, and fractions constituted 90.6 percent of all adult figuring.

From the survey it was found that only two and one half percent of all the problems in addition had more than four places in the largest addend. Almost all of the multiplication problems had either one or two place numbers in the multiplier. In subtraction the minuend which was most common was the three place minuend. In division 39.6 percent of all the problems had one number in the divisor; 43.4 percent had two place numbers in the divisor. The most commonly used fractions had denominators of halves, thirds, fourths, fifths, and eighths. "The further one goes in analysis of the processes as actually reported in this survey, the more convinced one is that life situations are much more simple than the problems represented in the old type of textbook. The work in accounts is very simple. Percentage involves simple discounts. The work in denominate

numbers does not involve a single case of reduction, ascending or descending, nor a single case of addition, subtraction, multiplication, or division of compound numbers. In fact denominate numbers merely come in as names for simple quantities and measures, bushel, foot, pound, and the others in common use. Nine-tenths of all figuring done by adults is done when money is involved."[1]

Woody[2] in his report on Types of Arithmetic Needed in Certain Types of Salesmanship says, "An interesting fact brought out by this investigation is the absence of the use of decimals except in connection with United States money. Denominate numbers as such are not used in the buying and selling of goods. Goods are sold in appropriate units and fractional parts thereof, but no attempt

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1. Wilson, Guy M., A Survey of the Social and Business Usage of Arithmetic, Contributions to Education, Number 100, Teachers' College, Columbia University

2. Woody, Clifford, Types of Arithmetic Needed in Certain Types of Salesmanship, Elementary School Journal, 1922

is made to reduce from one denomination to another. In the hardware store, if the gross was used no attempt was made to reduce the fractional portions of the gross to dozens. The same was true in regard to the foot and yard. No attempt was made to express the amounts purchased in terms other than yards and fractional portions thereof. No attempt was made to reduce an amount between fifty and one hundred feet in length to yards and feet, or to rods, yards and feet."

As a result of a study made in Denominate Numbers Used in the Factories of New Britain, Connecticut, V. Sala states[1]: "It is not profitable for children in the elementary grades to spend time on committing to memory tables of weights and measures. When the pupil understands the commodity, it is not difficult for him to apply the preferred unit of measurement to that commodity. The teaching of addition, subtraction,

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1. Sala, V., Denominate Numbers Used in the Factories of New Britain, Connecticut, Boston University, School of Education Thesis, 1931

multiplication, and division of compound denominate numbers has little value. Reductions ascending and reductions descending have no value in industry with perhaps the exception of the estimating department. They have, therefore, little justification as a school exercise."

"The subject of common fractions has undergone more change in the past twenty-five years than any other topic in arithmetic," says Clifford B. Upton[1]. Whereas a quarter of a century ago it was a frequent occurrence to ask pupils to add a series of fractions like 2/3 + 3/5 + 7/8 + 7/12, today such a problem is regarded as impractical and extreme. Over 94 percent of the common fractions found in present day life are those whose denominators are 2, 3, 4, 5, 6, 8, 10, 100, and sometimes twelve, but when it comes to genuinely practical problems in addition and subtraction of fractions we find that our range of

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1. Upton, Clifford B., Changing the Curriculum in Arithmetic, Teachers' College Record 28:341 - 359, December 1926

denominators is still more limited. There is frequent need for the addition of halves, fourths, and eighths but it is extremely difficult to find satisfactory problems where we are actually obliged to add halves and thirds, or fourths and sixths, or fractions having any other denominator than 2, 4, 8, or 16.

"Since our actual needs in the addition and subtraction of fractions are confined therefore to simple problems we are relieved of the teaching of a number of topics which formerly caused more or less trouble to pupils. Formal instruction in least common denominators, greatest common divisor, and least common multiple is no longer necessary, and these expressions need not even be mentioned. In adding fractions like 1/2 + 3/8 the common denominator is found by inspection and it is not serious if the common denominator found in this way is not the least common denominator.

"In the multiplication of fractions the main practical need is that of finding a fractional

part of a whole number, such as the cost of 3/4 pound of butter at so much a pound. This also includes the multiplication of a whole number by a mixed number such as 2 3/4 x 70¢. There is little or no need for the multiplication of a fraction by a fraction such as 3/4 x 5/6 as shown in the struggles of school textbooks to find suitable applications of this work. The case for the division of fractions is even less favorable. There may be an occasional use for division of fractions."

Carleton W. Washburne[1] says: "There is so little use for fractions that a mere knowledge of the basic principles will suffice in the case of most people. Only those knowledges and skills should be demanded of all children in our schools which will be used by most of them in life. We are, therefore, not justified in demanding as high a

1. Washburne, C.W., Superintendent of Schools, Winnetka, Illinois. Result of Investigation of Committee of Seven. Social Practices in Arithmetic Fundamentals, Elementary School Journal: 60, September 1926

standard in the manipulating of fractions as we have been requiring."

Research studies have also been made by Wise, Thorndike, Mitchell, Charters, Schorling and others. The conclusions reached were the same as those from similar studies previously mentioned.

As a result of the findings from the research studies in arithmetic in the Third Yearbook of the Department of Superintendence[1], the following list of processes is given for complete elimination: compound numbers, addition, subtraction, multiplication, division: greatest common divisor and least common multiple beyond the power of inspection; long confusing problems in common fractions; complex and compound fractions; reduction of denominate numbers; cases two and three in percentage; annual interest; compound interest, except savings; partial payments; true discount; proportion; ratio beyond the

1. Third Yearbook, Department of Superintendence, National Education Association of the United States, Washington, District of Columbia, page 41

ability of fractions to satisfy; partnerships with time; exchange, domestic, and foreign; apothecaries' weight; troy weight; table of folding paper, surveyor's table, table of foreign money; much of mensuration - trapezoid, trapezium, polygons, frustrums, spheres; cube root; the metric system.

The Committee on Arithmetic[1] deems it profitable to note the degree of difficulty indicated by the various studies made.

Degree of difficulty of addition problems

The typical addition example for the adult is the addition resulting from the purchase of two or three items at a grocery store. The amount for each item is usually less than one dollar. Therefore, we note for thorough drill and high speed and accuracy one, two, and three place examples, with two, three, four, and even five addends, for occasional drill and without speed requirements, there should be attention to larger examples -

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1. Fourth Yearbook, Department of Superintendence, National Education Association of the United States, Washington, District of Columbia, February 1926, pages 177 - 180.

even four, five, and occasionally six-place numbers, with addends running up to eight, ten, or occasionally as many as twenty: for informational attention no particular limit, but without any attempt at drill as such.

Degree of difficulty of subtraction problems

Research findings suggest drill on subtraction of one, two, three, and four place numbers with major emphasis upon two and three place numbers; occasional drill upon five and six place numbers; the informational usage for subtraction is relatively small.

Degree of difficulty of multiplication problems

Research findings suggest thorough drill on examples with one and two place multipliers: occasional drill with three and even four place multipliers. In drill work the multiplicand should seldom exceed three or four places.

Degree of difficulty of division problems

Ninety five percent of all division is covered by one, two, and three place divisors. Thorough drill on the same: occasional drill, very

occasional , with four place divisors.

Degree of difficulty of fractions

Confine thorough drill work to halves, thirds, fourths, fifths, eighths, tenths, and twelfths: occasional attention to sixteenths, twenty fourths, or any fraction involved in actual situations.

United States money and decimals

Little attention should be paid to decimals as such, buying and selling should be involved in fundamental processes from the beginning; hence, United States money should be constantly used from the beginning.

What shall be taught relative to denominate numbers? Tables as such are unnecessary. No reductions, either ascending or descending occur in business situations. Addition, subtraction, multiplication, or division of compound numbers are not needed.

Degree of difficulty of percents and percentage

There should be broad reading knowledge and general understandings of percents as they occur

in mark-down sales and general reading. The actual study of business situations will call for division of expenses, profits, loans, discounts, and so forth, and thus is introduced the opportunity for drill work on simple percentage and interest."

The actual analysis of usage has become the basis for inclusion of processes found useful in arithmetic, or the exclusion of processes not used. In analyzing and evaluating these tests in arithmetic the social utility theory is accepted.

Chapter I states the problem or task set for this thesis, and develops the basis for its study and solution.

Chapter II

Arithmetic Tests

a. Purpose of Testing

The task of examining the results of teaching is an important one. It has been facilitated by the use of standardized tests. Each teacher wishes to know the ability and weakness of her class in order to direct her work in such a way that the best results may be obtained. Tests will reveal whether the class is a homogeneous group or whether there is a sub-group that needs special treatment.

The main purpose of testing in arithmetic, however, is the diagnosis of pupil ability and pupil difficulties. The results of tests should be studied in the light of each pupil's individual attainments and weaknesses. When the weaknesses of individual pupils have been discovered, the pupils can be grouped according to common difficulties. Thus the child and his needs receive the full consideration of the teacher.[1]

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1. Wilson, G.M., and Hoke, K.J., How to Measure, Macmillan Company: New York, 1929, Preface

b. Kinds of Tests

Most of the standardized tests that have been prepared in arithmetic are of the survey type. The survey test is so constructed, theoretically, that a sample of each important element of subject matter is given. The examples are usually arranged in one of two ways, either in units of increasing difficulty or topically. When arranged in a series of increasing difficulty, the term scale is frequently applied, such as the Woody - Van Wagenen Scales. The purpose of the spiral arrangement is to determine the success or failure upon each significant element of subject matter. When used for diagnostic purposes they furnish data with reference to the samples used in the test. The main purpose of the survey test, however, is a quick inspection for grade classification of new pupils.

The teacher is concerned with details of instruction and with individual pupils. She needs detailed information in order to diagnose the needs of her pupils and to adjust her instruction to fit their particular needs. The teacher needs to know the reason for a pupil's poor work. His

difficulty may be in addition. This difficulty may be made specific by means of an inventory test. The term inventory is used because of the misapplication of the word diagnostic. A test that is truly diagnostic points out the specific difficulties of each child. Thus a teacher is able to discover whether a child fails in addition because he does not know his combinations, because his span of attention is too short, because he does not know how to carry, or for some other reason. The location of defects in individuals or in classes is one of the most important and helpful teaching procedures. Properly constructed tests may help in locating individual or class difficulties.

The necessity of considering the wide variety of types of examples in each process is clearly demonstrated by the results of a study by Overman[1] of the extent to which training on a few specific types of examples transfers to other types on which no training is given. He says: "The results

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1. Overman, J.P., An Experimental Study of the Effect of Methods of Teaching on the Transfer of Training in Arithmetic, Elementary School Journal, November 1930, Volume 31, pages 183 - 190

indicate that while transfer from one type of example to a related type may occur in large amounts, and may be complete in the case of some individuals, it is seldom complete for a group as a whole. This fact means that instruction and practice in the fundamentals of arithmetic must be based on full analysis of the fundamental processes. In view of the fact that transfer is seldom complete all the essential facts and all the essential steps in the processes should be taught."

"The present tendency in testing in arithmetic is to cover completely the operations in all of their specific phases and do this in such a manner that diagnosis of pupil weaknesses becomes relatively easy. That is, since we have now narrowed our work in arithmetic to the useful phases, it is unsatisfactory to ascertain merely the percentage of mastery by the use of a random sampling; what is wanted is a complete inventory of accomplishments and deficiencies."[1]

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1. Wilson, G.M., and Hoke, K.J., How to Measure, Macmillan Company, New York, 1929, Chapter IV, page 71

"A test to be genuinely diagnostic must be valid, that is it must cover all of the separate skills which are most important and which according to the aims of the course are to receive the most emphasis. --- Diagnosis will reveal the specific weaknesses and the use of remedial material will correct the defects without delay. In this way the pupil may be made aware of his own weaknesses and held largely responsible for overcoming them."[1]

An inventory test is valuable to the teacher in two ways: first as an instrument of diagnosing pupil needs and accomplishments, and second as a statement of the important types of examples in the fundamental processes. Ideally a series of inventory tests on the operations of arithmetic should guarantee an almost perfect measure of the specified part of the subject. Instead of presenting two or three of the fundamental

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1. Greene, H.A., and Jorgensen, Albert N., The Use and Interpretation of Educational Tests, Longmans, Green and Company, New York, 1929, Chapter IV, pages 39 - 40

combinations it should present all of them. This guarantees perfect validity because no elements of the field are left out of the test. The problem of reliability is also much simplified.

In reasoning or written problem tests the tendency is to choose problems that approximate life situations. "When arithmetic is put to practical business use, it is always connected with an actual situation. The problem then requires judgment or reason as to the process involved."[1] Problems should be meaningful to pupils. They should be based on actual life situations that the pupils understand. Failure in problem work has been due to the fact that the pupils did not know what it was all about; the problems involved obsolete business practices or situations entirely out of the experience or understanding of the pupil.

W.J. Osborn in his study of Diagnostic and Remedial Treatment for Errors in Arithmetical

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1. Wilson, G.M., and Hoke, K.J., How to Measure, Macmillan Company, New York, 1929, Chapter IV, page 106

Reasoning, after reviewing 30,000 errors made by 6,000 children on the Buckingham Problem Tests, Form I, states, "About 60 percent of the wrong answers are due to failure right in the beginning to understand what is to be done."

According to the Third Yearbook, "Isolated reasoning problems are not meeting the demands of life as it is. Under life conditions problems are solved when the individual is keen as to consequences. Any calculating is a small detail in a larger unified process."[1]

Details which make for interest insure that the child will identify himself with the problem. He will appreciate the critical situation in which affairs are, and will want to know what the answer is. "The problem must make an appeal. The situation must be understood, and if it is understood, the problem is half solved. It goes without saying that the problems must be expressed in language which children can understand, that

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1. Department of Superintendence, Third Yearbook, National Education Association, 1925, Chapter on Arithmetic, page 95

the computations should be within their powers and that the complexity should be within the maturity of their reasoning powers."[1]

The general recommendations growing out of an investigation on problems by Washburne[2] are as follows: "Problems should be so constructed as to present real situations familiar to the child. Children should be given many such problems to solve without special training in any generalized formal technique of analyzing problems. Concentration or practice in solving practical problems will yield gratifying results."

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1. National Society for the Study of Education, Report of the Society's Committee on Arithmetic, Twenty-ninth Yearbook, Public School Publishing Company, Bloomington, Illinois

2. Washburne, Carleton W., and Osborne, Raymond, Solving Arithmetic Problems, Elementary School Journal, Volume XXVII, December 1926, Number 4, page 296

Chapter III

Criteria for Selecting Tests

a. Major Criteria

"The major criteria relate to the ends which should be served by testing and which are more fundamental than the testing itself.

1. "The test should be in harmony with and reinforce the right curricular principles.

"This means that the true purposes of the subject from a curricular standpoint should be furthered by the test. In the fundamentals of arithmetic, speed and accuracy in automatic responses are wanted. The tests which measure progress in these lines are, therefore, directly reinforcing the tool purposes of arithmetic. ----

2. "A test should encourage, supplement, and reinforce proper methods of teaching.

"Since automatic memory results are wanted in arithmetic, the drill method is the appropriate one. A test, therefore, which calls for automatic mastery of the fundamental facts, properly reinforces the drill procedure.

3. A test should serve the true purposes of an examination.

a. "A good examination is the best teaching which can be done at the time.

b. "A good examination provides for a new view, a reorganization, or a worth-while application.

"If an examination is to be good teaching, it means that it will not be imposed from without but it will be under the direction of the teacher and fully in harmony with her plans and purposes. It means also that the test must be in harmony with good teaching and provide the kind of material that will mean new thinking in a worth-while situation.

"Regardless of a test's statistical and mechanical excellence, if these three criteria are not properly met the test is to be looked upon with extreme doubt, and unless for very special reasons, is to be rejected."[1]

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1. Wilson, G.M., and Hoke, K.J., How to Measure, The Macmillan Company, New York, 1929, pages 515 - 517

b. Minor Criteria

"A test may meet the three primary criteria and still not be nearly as good a test as possible. There are further refinements and these refinements have been the particular contribution of the scientific workers in the field of educational measurement."[1]

In choosing a standardized test make sure that the test really measures what it aims to measure. "The accuracy and detail with which a test is a measure of a trait, function, or school subject is called its validity."[2] For a test to possess validity it is necessary that the materials in the test be of prime importance, that the questions cover the essentials over which the complete mastery on part of pupils can be reasonably expected, and which they are given an opportunity to master in the course of study.

A standardized test should have at least two, preferably more duplicate forms, so a teacher may

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1. Wilson, G.M., and Hoke, K.J., How to Measure, Macmillan Company, New York, 1929, page 519

2. Simonds, P.M., Measurement in Secondary Education, Macmillan Company, New York, 1930, page 279

retest the class when she considers it desirable. Reliability refers to the correlation between the results of two forms of the same test. It is the accuracy of the test. When the judgment of the scorer enters into the reliability of the score, the test is unreliable, for the same test paper may be scored differently by two scorers or by the same scorer at different times.

A good test ought to be as easy to score as possible. Instructions for scoring, and an answer key which covers every possible response for which credit is given, should accompany every standardized test. In general, a test is more valuable in its interpretative results if it is valid, reliable, easy and economical to administer.

Most publishers include norms in the manual of directions accompanying the test. The final aim in fundamentals should be 100%. Thus, if the test is reasonable and right for the grade, the only acceptable norm is 100%. A norm is a standard based upon average performance. "Grade norms are less reliable and therefore less significant than

age norms."[1] The achievement in a grade for a test may give a norm that will be useful for the various schools using the same course of study. A norm made up from a group of schools similar in type may be used to compare the achievement in each of the schools concerned. Sometimes a number of cities agree to exchange results in a given test, and if the facts are known about course of study requirements, time allotments, and other pertinent data, interesting comparisons of achievement may be made.

"The teacher is urged, with increasing understanding, to become more critical of tests and thus aid in the general work of making them more acceptable; that is, more valid, accurate, reliable, and economical, and, at the same time, more nearly serving the larger aims of the subjects and of education in general."[2]

"Scientific testing in arithmetic has had

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1. Wilson, G.M., and Hoke, K.J., How to Measure, Macmillan Company, New York, 1929, page 536

2. Wilson, G.M., and Hoke, K.J., How to Measure, Macmillan Company, New York, 1929, page 5[illegible]1

wholesome effects upon the teaching of that subject. It has focused attention on the essential things and has supplemented admirably the curricular studies of Wilson, Wise, Woody, Charters, and others which call for a saner and wiser selection of subject matter on the basis of social usage."[1]

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1. Wilson G.M., and Hoke, K.J., How to Measure, Macmillan Company, New York, 1929, page 71

Chapter IV

Survey Tests

Survey tests are used to secure a general measure of the level of achievement of a school, or for quick pupil classification. They are not intended to provide information which will enable a teacher to locate the cause or the nature of the deficiency if the school or class is deficient.

Survey tests designed to measure achievement in a number of grades are usually available in two or three forms of equivalent difficulty. Each test contains sets of examples that have a wide range of difficulty, ranging from examples that are easy enough for pupils in the primary grades to others that are so difficult that only pupils in the upper grades can do them correctly.

The success of remedial teaching depends upon the accuracy and detail with which the specific skills involved in the processes of arithmetic are identified and isolated. Scientific investigations point to the fact that there are as many separate types of adding ability as there are different

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

types of examples to be added. This appears to be true not only for addition but for all of the operations with integers. The authors of standardized arithmetic tests differ in their ideas of the important process steps to be used in the operations of arithmetic. The more accurately a test measures the separate skills, the more helpful will that test be in revealing a picture of the specific weakness of a pupil. Some of the survey tests give rather detailed analysis of pupil difficulties.

In analyzing the tests the process steps used by the various authors are given. In the tables every fact met in every example in the fundamental processes is tabulated. Detailed analysis of tests follows.

a. The Cleveland Survey Tests[1] originated in the educational survey of the Cleveland Public Schools. Mr. Stuart A. Courtis of Detroit accepted an invitation from the Survey Staff to come to a conference at which the arithmetic tests were planned. The material was based on the Cleveland course of study.

The tests are composed of fifteen different sets of examples and are to be used in Grades III to VIII. They aim to test the fundamentals of arithmetic. There are four sets of the tests in addition; two in subtraction; three in multiplication; four in division; and two in fractions. In each of the fundamental processes and in fractions, the first set of examples is simple and each later test increases in difficulty. It is a timed test; the time covered by the test is twenty-two minutes. There are two forms I and II, similarly constructed.

<u>The Process Steps used in the Cleveland Survey Tests are</u>:

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1. <u>Cleveland</u> <u>Survey</u> <u>Tests</u> <u>in</u> <u>Arithmetic</u>, Forms <u>I and II</u>, Public School Publishing Company, Bloomington, Illinois

Addition.

1. Primary facts
2. One place columns, 5 addends carrying.
3. One place columns, 5 addends, zeros.
4. One place columns, 13 addends.
5. Four place figures, 5 addends, carrying.

Subtraction.

1. Primary facts.
2. Related facts.
3. One step borrowing.
4. Double borrowing.
5. Double borrowing with zero in minuend.

Multiplication.

1. Primary facts.
2. One place multiplier, 4 place multiplicand, carrying, requiring addition in higher decade.

Short Division.

1. Four or five digits in quotient no remainders.
2. Zero in quotient.

Long Division.

1. Involving chiefly the form of operation in long division: no carrying, in multiplication;

no borrowing in subtraction; no remainders.

2. Quotient difficulty.

   Trial divisor is not the correct quotient.

3. Quotient difficulty even though the increase by-one rule is used.[1]

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1. There are two rules for estimating the quotient in long division. First, by using the first digit of the divisor as it stands as the trial divisor. Second, for examples in which the second digit of the divisor is large, the first digit of the divisor increased by one is the trial divisor.

Table I Addition facts and their frequency.
Cleveland Survey Test Form 1.

	0	1	2	3	4	5	6	7	8	9
0	1	1		1	1		1		1	1
1	1		2	2	2	4	1	1	4	1
2	2	6	4	5	1	4	4	4		3
3		5		2	2	3	1	1	1	4
4	4	1	4	4	4	3	1	2	1	
5	1	1	1	3	3	5	2	1	2	1
6	3	2	3	1	1	3	2	4	2	2
7	1	2	1	1	3	4		3		1
8	1	3	2	4	1	5	4	3	5	4
9	1	2	4	1	3	3	1	3	4	5
10			1			1	2	3	4	1
11		2	1		4	2	2	1	4	
12	1	1		2	1	3	3	1	1	1
13	3			1	3		2	2	3	
14						1	2	2	1	3
15		3	4	1	1					1
16				1	5	1	2	1	2	3
17		1	2	1	2		3	3	3	3
18	1	1	3	2	4			3		
19	1	1	3	1			2	1		
20	1	1	1	6	3	1		1		2
21		1	2	2	1		2	2		2
22			2	1	1					
23		1	2	2			3		1	3
24		1	1	2		1			2	
25		1	2	2			2	1	1	1
26		1	1	3		2				1
27		1		1		2				
28				1		1				
29		1			2	1	1	1		
30			1			1				
31		2							1	
32			1	2		2	1		2	

	0	1	2	3	4	5	6	7	8	9
33	1	1		1				1		
34						1				1
35		1	1					1	1	
36							1	1		
37	1						2		1	
38					1					
39			1						1	
40		1			1		1			
41				1	1	1				
42		1	1							1
43		1	2		2	1			1	
44		1			1		1			
45	1	1			1	1		1	1	
46								1		1
47			1	1	1					
48		1								1
49						1	1	1		
50						1	1		1	
51							1			1
52					1			1		
53	1					1			1	
54								1		
55							1			2
56		1	1							
57		1	1					1		
58					1			2		1
59				1						
60			1							
61										
62					1					
63										
64		1								
65				1						

The numbers at the left are added to those across the top. These facts were found by adding up the columns. The number in the square indicates the frequency, for example 0 + 0 occurs once, 1 + 0 occurs once, 2 + 0 occurs twice and so on.
In Form 1, 90 of the primary addition facts, 143 of the higher decade facts to 39 + 9, and 59 of the higher decade facts used in carrying in multiplication are used. For a survey test the addition facts are well sampled.

Table II Addition facts and their frequency.
Cleveland Survey Arithmetic Tests Form 2

	0	1	2	3	4	5	6	7	8	9
0	1	1			1	1	1	1	1	
1	2		2	2	2	1	3	2		
2	2	7	1	4	4	7	1	3	6	5
3	1	5	1	2	5	3		1	4	1
4	1		2	3	2	3	1	1	1	3
5	2	1	3	1	1	3		2	2	1
6	1	1	3	1	3	1	2	2	2	2
7	2	5	2	3	5	2	4	1	3	6
8	2	2	5	1	1	1	3	3	4	2
9	2	2		3		3	2	3	3	3
10	1	2		5	1	3	5	2	2	1
11		2		2	2	2	1	1	2	1
12		1		1	3		1	1	1	2
13		2	1	2	2	3	1	1	2	
14	1	1	1	1	2	2	1		2	1
15	1	1	2	3	2	1	2	3	1	1
16		2	5	1	1	6	2	1	2	2
17	1		1	1	1	3		1	2	2
18	1	2	1	3	1	2	1	1	4	2
19			1		4	1	2	1	1	1
20		1			1	1	1		1	
21	1	3	1		1	2	3	2		1
22		1	1				2	2	1	2
23		2	2	2				1	1	
24	2			3	2		3			
25			2		1		2		1	1
26	1		1	2	1		1			
27		1	1	1				1		
28		1	1	1	1	1		1		1
29									1	
30			1	3	2	2	1	1		1
31			1							
32			1							

	0	1	2	3	4	5	6	7	8	9
33			2	1			2		1	
34							1	1		2
35		1		1				1		2
36						1			1	
37						1				
38		1								
39				2					1	
40			1							
41			1			1	1			
42		1	1				1	1		1
43					1			2		
44		1		1	1	1				
45							1			
46			1							
47								2	2	
48		1							1	1
49	1			1		2				
50					1					1
51				1	1					
52									1	
53		1								
54			1	1	1		2			1
55								1		1
56							1			1
57					1					
58						1				
59							1			
60					1					
63					1					
64						1				
65							1		1	
70										
71						1				

The figures on the left are added to those across the top. The figure in the square denotes the frequency. In Form 2, 89 of the primary addition facts, 162 of the upper decade facts, and 49 of the higher decade facts used in carrying in multiplication are used. The addition facts are well sampled for a survey test.

Table III Subtraction facts and their frequency.
Cleveland Survey Arithmetic Tests, Form 1.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0		1	1	1	1	2	1	1	1										
1		1	2	1	1	2			1	1	1								
2			1	1	1		2	1	1	2	2	1							
3					2	1	2	2	1			1	1						
4					1	1	2			2	1	2	1	1					
5						2	1	1	2		2	1	1	1					
6							1		1		1	1	1	2	1	2			
7								1		2	2	2	2	1		1	2		
8									1			1		2	2	2	2	1	
9										2	1	2	2			2	1	2	1

The figures on the left are subtracted from those across the top. The figure in the square denotes the frequency; for example 4 - 3 occurs twice, 5 - 3, once, 6 - 3 twice and so on.
79 of the primary subtraction facts are used. The subtraction facts are well sampled.

Table IV Subtraction facts and their frequency.
Cleveland Survey Arithmetic Tests, Form 2.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0		1	1		3	2			1	1									
1		1	2		1	1		1	2	1	1								
2				1	1	1	2	1	2			3							
3				1	2	2	1		1	1	1	1							
4						1	3	1		3	1	1	1						
5						1		1	1	3	2		2	1					
6							1		3		2	1	2	1		1			
7										1	2	2	2		1	1	3		
8											1			2	2	2	1	3	
9											1	1	2	2		2	4	1	

The figures on the left are subtracted from those across the top.
70 of the primary subtraction facts are used.

Table V Multiplication facts and their frequency.
Cleveland Survey Arithmetic Tests, Form 1.

	0	1	2	3	4	5	6	7	8	9
0	1		1	1	1	1	1	1	1	
1			1		1			1		1
2	1		3	3	2	3	4	1	3	1
3	1		2	3	3	2	3	2	3	3
4		1	2	4	3	2	3	3	3	3
5	1	1	2	3	3	3	4	2	3	4
6	1	1	3	2	4	4	2	4	2	3
7	1	1	3	3	4	4	3	4	3	2
8			3	3	3	2	3	2	3	3
9			3	2	2	1	2	2	3	2

The figures on the left are multiplied by those across the top. The figure in the square denotes the number of times the combination occurred.
85 of the multiplication facts are used.

Table VI Multiplication facts and their frequency.
Cleveland Survey Arithmetic Tests, Form 2.

	0	1	2	3	4	5	6	7	8	9
0	1	1	2		1		1	1	1	
1	1			1	1	1				
2		1	3	2	4	2	3	1	3	2
3			3	3	3	2	3	3	2	3
4	1		2	3	3	2	5	2	4	2
5			2	4	2	2	3	4	4	5
6		1	4	2	4	4	3	2	4	2
7	1	1	3	4	3	3	4	3	3	3
8	1		2	4	2	3	1	4	1	4
9	1		3		4	2	2	2	2	1

The figures on the left are multiplied by those across the top. The figure in the square denotes the frequency.
81 of the multiplication facts are used. The multiplication facts are well sampled in both forms.

Table VII Short Division facts and their frequency.
Cleveland Survey Arithmetic tests, Form 1.

	1	2	3	4	5	6	7	8	9
0	1	1	1		1	1	1		
1	2								
2	1								
3	1								
4		1		1					
5		1		1					
6		1	1			1			
7	1	1					1	1	
8	1	2	1	1		1		1	1
9			2						2
10		1			1				
12						1			
14		1							
15			1	1					
16			1	2				1	
17					1				
18		2	1			1	1		
20				1					
21			1				1		
22			1						
23						1			
24			1	2		1		1	
27			1						
28					1		1		

	1	2	3	4	5	6	7	8	9
29				1				1	
30					2				
32				1				1	
33									1
34				1		1		1	
36						1			1
38				1		1			
39									1
41					1				
42						1	2		
43						1			
48						1			
49							1		
50									1
51						1		1	
54						1			1
56								1	
63							2		2
64								1	
65								1	
72								1	1
78								1	
81									1
86									1

The figures at the top are the divisors, those at the left are the dividends.
91 of the short division facts are used in Form 1.

Table VIII Short Division facts and their frequency. Cleveland Survey Arithmetic Tests, Form 2.

	1	2	3	4	5	6	7	8	9
0	1			1					1
1	1								
2	1	1							
3	1		1	1			1		
4	1	1		1		1			
5		2		1	1				
6		2	1	1		2			
7	1		1				1		
8	1	1				1		1	
9			1						
10		1		1					
12		1		2		1			
13				1					
14							1		
15			1	1	1				
16		2		1				1	
18			1	1		1			
19			1						
20				1				1	
21				1			1		
23					1				
24			1			1			
25					1		1		
26								1	1
27			1						

	1	2	3	4	5	6	7	8	9
28				1			1		
29				1		1			
30					1	2			
34				1					
35					1				
36				1		1			
37							1		
40						1		1	1
42						1	1	1	
43									1
45						1			1
47					1				
48								1	
49						1	2		1
54									1
56							1	2	
58						1			
64								1	
69								1	
72									2
76									1
78								1	
85									1
88									1

The figures at the top of the table are the divisors, those at the left are the dividends. The figures in the squares denote the frequency. 94 short division facts are used in Form 2.

Table IX Number of digits in dividends and the divisors. Cleveland Survey Arithmetic Tests, Form 1.

	3	4	5
21	2		
22	2		
23	1		
31	3		
32	1		
33	1		
38			1
41	2	1	
42	1		
48			1
51		1	
52		2	
53		1	

	3	4	5
59			1
61		1	
62		1	
67			1
69			1
71		1	
72		1	
73		1	
78			1
81	1		
82		1	
88			1
97			1

The figures at the left of the table are the divisors, those across the top denote the number of digits in the dividends. The figure in the square denotes the frequency, for example 41 is used as a divisor three times, twice with a dividend of three digits, and once with a dividend of four digits.

Table X Number of digits in dividends, divisors, and their frequency. Cleveland Survey Arithmetic Tests, Form 2.

	3	4	5
21	2		
22	2		
23	1		
31	2	1	
32	1		
33		1	
38			1
41	2	1	
42	1		
48			1
51		1	
52		2	
53	1		

	3	4	5
59			1
61		1	
62		1	
67			1
69			1
71		1	
72	1		
73	1		
78			1
81		1	
82		1	
88			1
97			1

The figures across the top of the table denote the number of digits in the dividends, those at the left are the divisors. The figure in the square denotes the number of times each divisor is used with a 3, 4, or 5 digit dividend.

Table XI Denominators used in addition of fractions. Cleveland Survey Arithmetic Tests, Form 1.

	7ths	8ths	9ths	12ths	15ths	18ths
4ths						1
6ths					1	
7ths	3					
8ths		2		1		
9ths			5			

The fractions at the left are added to those across the top of the table. Denominators only indicated. The figures in the square denote the frequency, for example, sevenths are added to sevenths, three times.

Table XII Denominators used in addition of fractions. Cleveland Survey Arithmetic Tests, Form 2.

	4ths	5ths	6ths	7ths	8ths	9ths
5ths		1				
7ths				3		
8ths					2	
9ths						6
10ths					1	
14ths	1					
21sts			1			

The fractions on the left are added to those across the top of the table. The denominators only are indicated. The figures in the squares denote the frequency.

Table XIII Denominators used in subtraction of fractions. Cleveland Survey Arithmetic Tests, Form 1.

	5ths	7ths	8ths	9ths	10ths	14ths	21sts
4ths						1	
5ths	1						
7ths		3					
8ths			2		1		1
9ths				6			
10ths			1				

The fractions at the left are subtracted from those across the top of the table. Denominators, only, are indicated.

Table XIV Denominators used in subtraction of fractions. Cleveland Survey Arithmetic Tests, Form 2.

	5ths	7ths	8ths	9ths	15ths	18ths
4ths						1
5ths	1					
6ths					1	
7ths		3				
8ths			2			
9ths				6		
12ths			1			

The fractions at the left are subtracted from those across the top. Denominators only indicated.

Table XV Fractions used in multiplication. Denominators only are indicated. Cleveland Survey Arithmetic Tests, Forms 1 and 2.

Form 1				Form 2			
	6ths	10ths	20ths		6ths	12ths	21sts
5ths	1			6ths			1
6ths		1	1	8ths		1	
				15ths	1		

The fractions on the left are multiplied by those across the top of the table.

Table XVI Fractions used in division. Cleveland Survey Arithmetic Tests, Forms 1 and 2. Denominators only are indicated.

Form 1				Form 2		
	6ths	12ths	21sts		6ths	10ths
6ths			1	5ths	1	
8ths		1		6ths		1
15ths	1			20ths	1	

The fractions on the left are divided by those across the top of the table.

Table XVII Number of facts used in the Cleveland Survey Arithmetic Test.

Forms	100 Primary Addition Facts	300 Upper Decade Facts to 39+9	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
1	90	143	59	79	85	91
2	89	162	49	70	81	94

The following appear to the author as pertinent criticisms.

Some of the examples go beyond social usage.[1]

1. Addition of whole numbers

   One place columns with thirteen addends.

2. Fractions

   Addition of sixths and fifteenths; of fourths and eighteenths.

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1. Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin Company, Boston, Massachusetts, 1926, Chapter XIV, page 122

1. Department of Superintendence, Fourth Yearbook, 1926, Chapter VII, pages 173 - 220

Subtraction of sixths from twenty-firsts.

Division of sixths and fifteenths; sixths and twenty-firsts. Division of fractions is seldom if ever used in business or social usage.

The foregoing tables show that the Cleveland Survey Tests cover the facts in the fundamental processes fairly well, and give some analysis of pupil difficulties. By omitting Set J, which contains thirteen addends, and Set O which contains fractions, the tests can be made to conform to the newer requirements in arithmetic based on social usage.[1] These tests originated in the survey of the Cleveland Public Schools and the material was based on the Cleveland course of study. The survey was made before the new course of study went into operation; therefore some of the examples go beyond social usage.[2] If Set J and Set O are omitted the Cleveland Survey Tests may be profitably used for a general survey or for pupil classification.

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1. Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin Company, Boston, Massachusetts, 1926, page 122

2. See Measuring the Work of the Public Schools, Cleveland Education Survey, The Survey Committee of the Cleveland Foundation, Cleveland, Ohio , 1916

b. "The Compass Survey Tests in Arithmetic[1] are designed to cover an adequate sampling of the arithmetic skills commonly taught in grades 2 to 8 of the elementary school."[2] The examinations are prepared in two forms, A and B. The Elementary Examination prepared for Grades 2, 3, and 4 consists of tests in addition, subtraction, multiplication, and division of whole numbers. The Advanced Examination designed for Grades 4 to 8 consists of examples in whole numbers, fractions, decimals, denominate numbers, percentage, and general problems.

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1. Compass Survey Tests in Arithmetic, Greene, Knight, Ruch, Studebaker, Scott, Foresman and Company, New York, 1927

2. Greene, H.A., Knight, F.B., Ruch, G.M., Studebaker, J.W., Manual of Directions for Compass Survey Tests in Arithmetic, Scott, Foresman and Company, Chicago, Illinois, 1927, page 1

The following Process steps are used in the Compass Survey Tests, Forms A and B:

Process steps used in addition

1. Primary combinations.
2. Decade combinations.
3. Short columns carrying sums to 30.
4. Columns, carrying, gaps, zeros.

Process steps used in subtraction

1. Primary combinations.
2. Simple subtraction without borrowing with zero in answer.
3. One step borrowing.
4. Double borrowing.
5. Borrowing, vanishing lefts.
6. Quadruple borrowing with nines in subtrahend.

Process steps used in multiplication

1. Primary facts.
2. Zero primary facts.
3. One place multiplier, no carrying, zero in multiplicand.

4. One place multiplier, zero in multiplicand.
5. Two place multipliers, carrying.
6. Three place multiplier, zero in multiplicand.
7. Single zero in multiplying.
8. Four place multiplier, carrying, zero, and four place multiplicand.

Process steps used in division

1. Two place divisor, no carrying in multiplication.
2. Two place divisor, six place dividend, borrowing in subtraction, remainder.
3. Two place divisor, five place dividend, borrowing, carrying, no remainder.
4. Three place divisor, four place dividend, borrowing in subtraction, carrying in multiplication, no remainders.

The Advanced Examination, Grades 4 - 8, Form A and B of the Compass Survey Tests in Arithmetic consist of examples in addition, subtraction, multiplication, and division of whole numbers, decimals, fractions, and denominate numbers; work in percentage and general problem solving.

Table XVIII
Addition facts and
their frequency. Compass
Survey Arithmetic Tests.
Elementary Examination

Form A

	0	1	2	3	4	5	6	7	8	9
0	1		1				1	1		1
1	1	2	1	1	1	1		1	1	
2			1			1	1	1		
3	1	1	1	1	1		1			
4		1		1				1	1	
5			1		1	1		1	1	
6	1	1			1	1	1	1		1
7	1		1	1		1	1	1	1	1
8	1	1	1	1	1					
9		1		1	1	1	1			1
10						1		1	1	
11										
12								1		1
13						1	1		1	1
14									1	
15								1		1
16										
17										1
18						1				
19						1		1		
20										
21										
22									1	
23										1
27										
29				1						
38							1			
45								1		
54										
56					1					
64									1	
72							1			

Table XIX Addition
facts and their frequency.
Compass Survey Arithmetic
Tests. Elementary Examination.

Form,B

	0	1	2	3	4	5	6	7	8	9
0		1		1	1	1			1	
1		2		1	1		1			
2	1	1		1	1	1	1		2	1
3						1		1	1	1
4	1		1		1	1	1		1	1
5	1	1		1			1			1
6			1	1					1	1
7		1			1					1
8			1			1	1	1	1	1
9	1		1					1	1	
10	1		1	1		1	1			
11					1			1		
12						1				
13					1			1		
14						1		1		1
15							1			
16					1			1		
17							1	1	1	
18								1		
19							1		1	
20									1	
21						1				
22										
23										
27						1				
29										
38								1		
45										
54								1		
56										
64						1				
72									1	

The numbers on the left are added to those across the top of the table.
58 of the primary addition facts, 20 of the upper decade facts, and 4 of the higher decade facts are used in Form A. 50 of the primary addition facts, 26 of the upper decade facts, and 3 of the higher decade facts, are used in Form B.

Table XX Subtraction facts and their frequency. Compass Survey Arithmetic Tests. Elementary Examination.

Form A

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	1		1	1			1											
1		1	1	1	1		1		1		1								
2				1		1		1	1	1		1							
3				1		1		1			1								
4					1			1			1	1		1					
5						1	1						1	1					
6								1	1	1	1				1	1			
7										1			1	1			1		
8										1	1	1	1	1				1	
9										2	1			1	1	1		1	1

The numbers on the left are subtracted from those across the top of the table.
In Form A, 54 subtraction facts are used.

Table XXI Subtraction facts and their frequency. Compass Survey Arithmetic Tests. Elementary Examination.

Form B

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1		1			1	1		1	1									
1						1		1		1	1								
2		1		1		1				1									
3					1		1	1		2		1	1						
4					1	1	1			1	1			1					
5						1		1	1	1	1	1			1				
6							1					1	1	1					
7								1	1		1			1	1	1			
8														1	1	1	1	1	
9										1		1	1	1	1		1		

The numbers on the left are subtracted from across the top of the table.
In Form B, 54 subtraction facts are used.

Table XXII Multiplication facts and their frequency. Compass Survey Arithmetic Tests. Elementary Examination.

Form A

	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	2	2	1
9	1	1	1	1	1	1	1	1	2	1

Table XXIII Multiplication facts and their frequency. Compass Survey Arithmetic Tests. Elementary Examination.

Form B

	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	2	1
9	1	1	1	1	1	1	2	1	2	1

The numbers on the left are the multipliers.
All the multiplication facts are used in Forms A and B.

Table XXIV Short division facts and their frequency. Compass Survey Arithmetic Tests.

Form A

	1	2	3	4	5	6	7	8	9
0	1				1			1	
2		1	1						
3								1	
4				1				1	
5	1								
6		1	1			1			
7			1				1		
8				1					
11		1							
12						1			
14		1	1						
15			1						
16				1				1	
18		1							
19							1		
21			1						
24								2	
27				1					
32								1	
34				1	1				
35						1			
37						1			
40								1	
42							1		
44					1				
45					1				
54						1			
55							1		
60							1		
63									1
75								1	
84									1

Table XXV Short division facts and their frequency. Compass Survey Arithmetic Tests.

Form B

	1	2	3	4	5	6	7	8	9
0		1		1		1			
3	1						1		
4	1	1							
5			1						
6									1
7					1				
8		1							
9			1						1
10					1				
13		1						1	
15				1					
16		1	1						
18			1						1
20				1					
22					1	1	1		
24			1						
25					1				
27							1		1
29						1			
30				1					
33								1	
35							1		
36				1					
37					1				
42						1			
46									1
53						1			
54								1	1
56								1	
63									1
67							1		
68									1
76									1

The figures at the top are the divisors. They are divided into the figures at the left.
42 short division facts are used in Form A, 46 in Form B.

Table XXVI Addition facts and their frequency. Compass Survey Arithmetic Tests. Advanced Examination

Form A

	0	1	2	3	4	5	6	7	8	9
0						1				
1	1		1	1		2	3			
2	1	1			1	2	1	1	1	1
3		1			1			1	1	
4				1	1		1			
5	1			2		1				
6				1		1		1		
7			1		1			1	1	2
8		1			1	1		1	1	
9		1					1	1		
10						1		2		
11		1			1					1
12			2							
13	1									1
14			1							
15			1					2	2	1
16							2		1	1
17					1					
20					1					
21			1							
22				1						
24							1			
25	1	1				1				
26										1
37										1
38										

Table XXVII Addition facts and their frequency. Compass Survey Arithmetic Tests. Advanced Examination.

Form B

	0	1	2	3	4	5	6	7	8	9
0										
1	1	1		1	1	1		1	2	
2		1		2	1	1			1	
3				1		2	1		1	
4		1								
5	1	2			1	2		1	1	
6		1	1			1	1	1	1	
7						1				
8	1						1		1	
9			1	2		1			1	1
10										1
11	1							1	1	1
12				1			1		1	
13	1								1	
14						1			1	1
16								1		
17			1							
18	1		1				1			
19							1	1	1	
20						1		1		
21										1
22					1					1
23	1									1
26							1			
27							1			
39									1	

Numbers on the left are added to those across the top of the table.

40 of the primary addition facts, and 28 of the upper decade facts are used in Form A.

38 of the primary addition facts, and 31 of the upper decade facts are used in Form B.

Table XXVIII Subtraction facts and their frequency.
Compass Survey Arithmetic Tests, Advanced Examination.

Form A

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0																			
1		1	1																
2				1	1														
3					1				1	1									
4					1			1											
5						1	1		1	1	1								
6									1			1							
7												1		1					
8									1	1			1	1					
9										1	1				1		1		

26 subtraction facts are used in the Advanced Examination, Form A.

Table XXIX Subtraction facts and their frequency.
Compass Survey Arithmetic Tests, Advanced Examination.

Form B

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0																			
1				1															
2				1				1		1									
3				1			1		2										
4						1	1			1	1								
5										1	1								
6								1			1			1					
7										1	1	1							
8											1	1							
9													1					1	1

24 subtraction facts are used in the Advanced Examination, Form B.

Table XXX Multiplication facts and their frequency. Compass Survey Arithmetic Tests. Advanced Examination.

Form A

	0	1	2	3	4	5	6	7	8	9
0	1	1	1			1	1			
1		1	1		1	1	1	1		1
2	1	1		1		1				
3			1		1					
4			1							1
5	1		1		1		1	1		
6	5		1	1	2	1		2		
7		1	1			1	1			1
8										
9			1							

Table XXXI Multiplication facts and their frequency. Compass Survey Arithmetic Tests. Advanced Examination.

Form B

	0	1	2	3	4	5	6	7	8	9
0	2			2			1		2	
1	1	1	1	1	2			2	1	
2	1	1					1		1	
3	1	1		1	1		1	1	2	
4		1			1					
5	2	1			1	1	1		1	2
6	1			1			1		1	
7										
8	1	1		1				1	1	
9			1					1		

37 multiplication facts are used in Form A.
42 multiplication facts are used in Form B.

Table XXXII Fractions used in addition. Compass Survey Arithmetic Tests. Advanced Examination,

Form A

	2/5	2/7	1/12	1/15
2/3			1	
2/3	1			1
7/9	1			
9/14		1		

Table XXXIII Fractions used in addition. Compass Survey Arithmetic Tests. Advanced Examination,

Form B

	1/3	2/3	1/4	5/8	3/10
1/2					1
1/2		1	1		
1/5				1	
4/9	1				

Fractions at left are added to those across the top of the table.

Table XXXIV Short Division facts and their frequency. Compass Survey Arithmetic Tests. Advanced Examination.

Form A

	2	3	4	7
3	1			
8			1	
10		1		
20	1			
24			1	
30				1
45				1

Table XXXV Short division facts and their frequency. Compass Survey Arithmetic Tests. Advanced Examination.

Form B

	3	4	6	8
4	1			
6			1	
9		1		
10		1		
21	1			
24			1	
25				1
26		1		
28		1		
50				1

The figures at the top are the divisors. 7 short division facts are used in Form A and 10 in Form B, in the Advanced Examination.

Table XXXVI Fractions used in subtraction. Compass Survey Tests. Advanced Examination. Form A.

	1 1/4	7/8	4/9
1/6		1	
1/6			1
7/12	1		

The fractions on the left are subtracted from those across the top.

Table XXXVII Fractions used in subtraction. Compass Survey Tests. Advanced Examination, Form B.

	1 1/5	5/6	7/8
1/4		1	
1/5			1
9/15	1		

The fractions on the left are subtracted from those across the top.

Table XXXVIII Fractions used in multiplication and their frequency. Compass Survey Tests. Advanced Examination. Form A and B.

1/3	1
2/3	2
1/4	1
3/4	2
2/5	2
3/5	1
4/5	2
5/6	2
3/8	2
7/8	1
7/12	1
7/15	2

Table XXXVIII is read as follows: 1/3 is used once; 2/3 twice and so on.

Table XXXIX Decimals used in Compass Survey Advanced Examination. Forms A and B.

Hundredths	+	hundredths
Hundredths	-	hundredths
Thousandths	+	thousandths
Tenths	x	tenths
Hundredths	x	whole numbers
Hundredths	÷	tenths
Hundredths	÷	whole numbers
Millionths	÷	whole numbers
Whole numbers	÷	hundredths

The same combinations of decimals are used in both forms.

Table XL Divisors and number of digits in dividends. Compass Survey Advanced Examination.

Form A			Form B		
	4	5		4	5
21	1		22	1	
36	1		38	1	
65		1	56		1

The figures at the top of the table denote the number of digits in the dividends. The figures at the left are the divisors.

Table XLI Types of examples in percentage.
Compass Survey Tests. Advanced Examination.

Type of Example	Frequency Forms	
	A	B
Changing fractions to percents	2	2
Changing decimals to percents	2	2
Changing percents to fractions	2	2
Changing percents to decimals	2	2
Finding a certain percent of a number	3	3
Finding a number when a certain percent of it is given	2	2
Finding what percent one number is of another	2	2
Problem: Given the selling price in money and percent in relation to cost. Find the cost.	1	1

Table XLII Situations Involved and processes used in general problems, Compass Survey Advanced Examination. Forms A and B.

Situation Involved	Processes Used
Cost of candy bars	Multiplication
Making change	Multiplication and subtraction
Finding average of numbers	Addition and division
Finding area of rectangle	Multiplication
Finding interest	Multiplication
Finding commission	Addition and Multiplication

Table XLIII Number of Facts Used in the Elementary Examination of the Compass Survey Tests in Arithmetic

100 Primary Addition Facts		300 Upper Decade Facts		80 Higher Decade Facts		100 Primary Subtraction Facts		100 Multiplication Facts		Short Division Facts	
Form		Form		Form		Form		Form		Form	
A	B	A	B	A	B	A	B	A	B	A	B
58	50	20	26	4	3	54	54	100	100	42	46

The following appear to the author as pertinent criticisms.

The Compass Survey Tests in Arithmetic are designed to test material in grades 2, 3, 4 that the newer curriculum in arithmetic recommends for higher grades.[1] The following unreasonable grade requirements in grades 2 and 3 were noted:

1. Consult Fourth Yearbook, Department of Superintendence, National Education Association, 1926, Chapter on Arithmetic, page 177

Formal drill in grade two.

Addition of three place figures with four addends.

Addition of five addends, 3 and 4 place figures, gaps, zeros.

All the facts in multiplication.

Three and four place figures in the multiplicand, three and four figures in the multiplier, zero difficulties.

Five place figures in the subtrahend and four figures in the minuend in subtraction.

Examples in short and long division.

The elementary examinations of the Compass Survey Tests in Arithmetic do not conform to the recommendations of the forward looking educators of the day and are therefore not to be recommended for use in grades 2, 3, and 4, They do not adequately test the facts in the fundamental processes and are therefore of less value in the upper grades than other survey tests in the same field.

Table XLIV Summary of Facts Used in the Advanced Examination of the Compass Survey Tests in Arithmetic.

100 Primary Facts		300 Upper Decade		80 Higher Decade		100 Primary Subtraction		100 Multiplication Facts		Short Division Facts	
Form		Form		Form		Form		Form		Form	
A	B	A	B	A	B	A	B	A	B	A	B
40	38	28	31	0	0	26	24	37	42	7	10

From the foregoing tables it may be observed that the Advanced Compass Survey Tests in Arithmetic do not give an adequate sampling of the facts in the fundamental processes.

The following violations of the recommendations of the Committee on Arithmetic, Fourth Yearbook, Department of Superintendence, National Education Association were noted:

1. Unreasonable grade expectations, percentage interest and commission in grades four and five.

2. The examples in fractions go beyond social usage:

Addition of ninths and fifths, fourteenths and sevenths.

Subtraction of fifteenths and eighths, ninths and thirds.

Division of fractions.

The examples in percentage involve meaningless practice in work of doubtful value according to recent studies.[1]

The Compass Survey Tests in Arithmetic have little to recommend them. They are not in harmony with nor do they reinforce the right curricular principles; they do not test adequately the facts in the fundamental processes.

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1. Wilson, G.M., A Survey of the Social and Business Usage of Arithmetic, Contribution to Education No. 100, Teachers' College, Columbia University

Wise, C.T., Arithmetic Problems Arising in Various Occupations, Elementary School Journal, October, 1919, page 118

Woody, C., Types of Arithmetic Needs in Certain Types of Salesmanship, Elementary School Journal, Volume 22, page 505 - 520, March 1922

c. The Compass Diagnostic Tests[1] in Arithmetic do not meet the standards of inventory tests and therefore do not come under that classification.

The Compass Diagnostic Tests consist of twenty different tests each dealing with some one of the processes or with problem solving. The method of diagnosis is to give a test in a single process to determine the level at which the skill of a pupil in an element of the process breaks down, and thus to make it possible for the teacher to locate cause of weakness in the process.

The following process steps are used in the Compass Diagnostic Tests.

Addition

1. Primary facts.
2. Decade drill.
3. Single column addition, carrying.
4. Single column addition, carrying, zeros.
5. Two or three place figures, carrying, no zeros.

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1. Compass Diagnostic Tests in Arithmetic, Scott, Foresman and Company, Chicago Illinois, 1925

6. Two or three place figures, carrying, with zeros.
7. Columns, gaps, zeros.

Subtraction

1. Primary facts.
2. Simple subtraction without borrowing.
3. Simple subtraction without borrowing with zero in the answer.
4. One step borrowing.
5. Double or triple borrowing.

Multiplication

1. Primary facts.
2. One place multiplier, no carrying.
3. One place multiplier, carrying.
4. One place multiplier, zero in multiplicand.
5. Two, three, and four place figures in multiplicand and multiplier.

Short Division

1. Primary facts.
2. Three or more digits in quotient, no carrying, no remainder.
3. Two or more digits in quotient, carrying, remainders.

4. Zero in quotient.

Long Division

1. Involving chiefly the form of operation in long division; divisor of two places and its right hand figure very small, no carrying in multiplication; no borrowing in subtraction.

2. Two place divisor, four place dividend, carrying, borrowing, remainder, but no quotient difficulty.
3. Quotient difficulty, trial divisor is not the correct quotient.
4. Quotient difficulty even though the increase by one rule is used.[1]

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1. There are two rules for estimating the quotient in long division. First, by using the first digit of the divisor as it stands for trial divisor, second, for examples in which the second digit is large, the first digit of the divisor increased by one is the trial divisor.

Table XLV Addition facts and their frequency.
Compass Diagnostic Tests in Arithmetic. Form A

	0	1	2	3	4	5	6	7	8	9
0	1		2		2	3			1	1
1	1	11	2	2	4		4	3		2
2	5	3	6		6	3	3	3	2	10
3			4	2	2	3	4	2	3	2
4		3	2	5		1		2	1	1
5			3	5	2			5	1	3
6		4	3		1		2	5	3	3
7	5	3	1	6	4	2	2	4	1	7
8		5	4	1		1		2	5	
9	2	8	1	2	4	5	1			5
10	3		3	2	5	3		1	1	1
11	1	2					1	5		6
12	5		2		5		6	3		
13							4	2	5	
14		5	3					5		
15								1	1	7
16				1	2		8	1	4	
18		2		4		1	1	4	4	
19	2	1	2		2			2		

	0	1	2	3	4	5	6	7	8	9
20	1		2	1	5		1	2		2
21		4			7					
22	3				1		8	1		
23				2						4
24		3	2					7		1
25			4					7		
26			4		1		3	2		
27										3
28						3		2		
29										2
30			1							
31	3									3
32			1				2			
33				1		2				
36			2							
38								2		
40			2							
42					2					

The figures on the left are added to those across the top of the table.
75 primary addition facts, 81 upper decade facts and 2 higher decade facts are used.

Table XLVI Subtraction facts and their frequency.
Compass Diagnostic Tests in Arithmetic. Form A.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	3	2	2				4	2	4	2								
1		16	2					3										
2			8	2		3	3		2		2							
3				4	2		3	2			3							
4					5	2	1	3	3	3	3	2		2				
5						4			4			2	2					
6							4	3		4	4	3	2	2				
7								[illegible]	3		4			2	2			
8									4		4	2	2			2		
9										2			4	1	3	2	3	3

The figures at the left are subtracted from those across the top. 58 subtraction facts are used.

Table XLVII Multiplication facts and their frequency. Compass Diagnostic Tests. Form A.

	0	1	2	3	4	5	6	7	8	9
0	9		3	8	5	4	4	2	5	4
1	4		6		4	2	6	2		9
2	7	1	4	1	5	2	2	2	3	3
3	4	2	3	3	5	2	2	2	3	3
4	2		4			1	3		1	7
5	5		3	1	4		2		3	4
6	6		4		5		2	3	4	4
7	6	1		6		6	3		6	3
8	3		1		4	1			4	1
9	1		3	5	3	1	2		3	1

The figures at the left are the multipliers.
77 facts in multiplication are used.

Table XLVIII Short Division facts and their frequency. Compass Diagnostic Tests, Form A.

	0	1	2	3	4	5	6	7	8	9
0									1	
2			1							
3	1				1					
4	3	1								
5						3				
6			1	2						
7								1		
8			1		5	1				
9				2						1
10						1			1	
14							1			
16				1						
18			1							
22				1						
27				1	1		1			
28					2		1		1	
29			1							

	0	1	2	3	4	5	6	7	8	9
32									1	
35						1				
36				3		2				
41						1				
42						1				
44						1	1	1		
45					1					
48								1		
49										
52						1				
54						1				
56							1			
62										1
63								2		1
65									1	
72									1	1
81										2

The figures at the top are the divisors. 49 short division facts are used.

Table XLIX Divisors and number of digits in dividends. Form A.

	40	48	63	64	74	78	84	206	266	296	406	408	560	627	890
4			1	1											
5	1	1			1	1	1						1		
6									1		1	1		1	1
7								1		1					

The figures across the top are the divisors, those at the left are the number of digits in the dividends.

Table L Fractions to be changed to equivalent Forms. Compass Diagnostic Tests in Arithmetic, Form A.

1/10 = /4	3/4 = /44	1/8 = /24
1/2 = /6	2/5 = /35	5/8 = /40
1/2 = /10	2/5 = /40	4/9 = /18
1/2 = /12	3/5 = /20	4/9 = /36
1/2 = /14	4/5 = /15	1/10 = /20
1/2 = /20	4/5 = /30	3/10 = /30
1/2 = /30	1/6 = /12	9/10 = /50
1/3 = /12	1/6 = /18	1/11 = /77
1/3 = /15	1/6 = /30	6/11 = /44
1/3 = /24	5/6 = /18	1/12 = /36
2/3 = /6	5/6 = /24	5/12 = /24
2/3 = /24	1/7 = /77	1/15 = /30
1/4 = /16	2/7 = /14	2/15 = /60
1/4 = /20	6/7 = /35	3/20 = /60
3/4 = /12	1/8 = /16	16/25 = /50

Each fraction is to be changed to its equivalent form using the given denominator.

Table LI Fractions to be changed to lowest terms. Compass Diagnostic Tests in Arithmetic. Form A.

5/4	8/10	15/12	10/18	20/20	21/24	41/40
7/6	8/12	12/14	23/18	28/20	31/24	57/44
8/8	13/12	25/15	19/19	14/21	47/35	80/50

Table LII Groupings of fractions for which common denominators are to be found. Compass Diagnostic Tests in Arithmetic. Form A.

1/2 + 1/12	1/7 + 1/2 + 1/14
1/2 + 1/5	1/8 + 1/5
1/2 + 1/4 + 1/5	1/10 + 1/2
1/3 + 1/2	1/12 + 1/9
1/3 + 1/8 + 1/12	1/12 + 1/12
1/3 + 1/5 + 1/15	1/15 + 1/10
1/3 + 1/4 + 1/6	1/15 + 1/20
1/4 + 1/2	1/16 + 1/4 + 1/8
1/4 + 1/11	1/17 + 1/17 + 1/17
1/4 + 1/10	1/19 + 1/19
1/6 + 1/18	1/21 + 1/21 + 1/21
1/6 + 1/5 + 1/2	1/35 + 1/7 + 1/5
1/7 + 1/11	1/50 + 1/25 + 1/10

In Test VI Subtraction of Fractions and Mixed Numbers the following skills are tested:

1. Changing fractions to equivalent forms
2. Finding common denominators
3. Expressing mixed numbers as improper fractions
4. Reducing answers to lowest terms and finding errors

The fractions are those used in subtraction of fractions.

Table LIII Fractions and mixed numbers used in subtraction. Compass Diagnostic Tests in Arithmetic, Form A.

1/2 - 1/3	9 1/6 - 4 5/9	11/12 - 5/8
7 1/2 - 5 1/2	10 5/6 - 1/5	7/12 - 1/3
9 1/2 - 7 3/5	4/7 - 3/7	7/12 - 1/4
6 1/2 - 1 9/14	5 2/7 - 4/7	3 1/14 - 2 3/7
6 1/3 - 2 3/4	8 2/7 - 1/2	9/16 - 5/16
7 1/3 - 3 2/3	7 3/8 - 5/6	8 11/30 - 11/30
7 1/3 - 6 8/15	16 1/9 - 7	4 - 3 3/4
2 1/4 - 3/8	6 2/9 - 4 7/12	10 - 8/9
6 3/4 - 3 1/6	8 4/9 - 1/3	6 - 4 7/12
4 5/6 - 1/2	1 10/11 - 7/11	9 - 8 13/20

In multiplication of fractions and mixed numbers the following skills are tested:

1. Changing mixed numbers to improper fractions.
2. Cancellation in multiplication of fractions.
3. Reducing answers to best form.

Table LIV Fractions and mixed numbers used in multiplication. Compass Diagnostic Tests in Arithmetic, Form A.

3 1/2 x 1 1/2	2 4/5 x 1/2 x 5/6	5/9 x 3/10
1 1/2 x 2 x 1/3	1/6 of 2	9/11 of 2/3 of 7
1/2 of 1 1/8 x 1/2	2 1/6 x 1 1/2	1/14 of 3
1 1/2 x 1/8 x 2 1/2	1/6 x 9/11	1/16 x 8/25
2 1/2 x 0	5 5/6 x 3 3/5	2 2/21 x 9/ 22
1/3 x 1/9 x 1/3	1 1/7 x 5 1/4	1 1/24 x 1 4/5
1 2/3 x 2/3 x 1 4/5	3/7 x 4/5	1 1/27 x 1 2/7 x 11
3/4 x 3 x 1/2	7/8 x 3	2 x 1/3
4 3/4 x 15	3/8 x 12	10 x 1/3 x 1/4
2/5 x 1/6	4/9 x 1/4 x 4/9	4 x 5/6

The following skills are tested in Test VIII: Division of fractions and Mixed Numbers Form A.

1. Changing mixed numbers to improper fractions.
2. Cancellation in division of fractions.
3. Reducing answers to best form.
4. Changing from multiplication to division form

The same fractions are used in the preliminary examples testing the above skills as are used in the division examples.

Table LV Fractions and mixed numbers used in division, Compass Diagnostic Tests in Arithmetic, Form A.

1/2 ÷ 2/3	3 1/5 ÷ 1 1/2	2 1/12 ÷ 12/25
10 1/2 ÷ 1 1/6	4 1/6 ÷ 7 1/2	3/16 ÷ 9/40
1 1/2 ÷ 15	1 1/6 ÷ 3/5	7/16 ÷ 3/8
3 1/2 ÷ 2 1/10	3/7 ÷ 2	1/18 ÷ 7/9
2 1/3 ÷ 2 1/2	3 3/7 ÷ 2 1/2	1/20 ÷ 3/8
1 1/3 ÷ 1 1/15	4/7 ÷ 6/7	19/24 ÷ 3 2/3
1 1/3 ÷ 1/2	5/7 ÷ 1/2	4 ÷ 1/2
2/3 ÷ 3/11	6/7 ÷ 3/4	15 ÷ 1 1/2
3/4 ÷ 9/16	3/8 ÷ 9	10 ÷ 1/3
1/5 ÷ 1/2	3/8 ÷ 2 1/4	9 ÷ 5 1/4

Table LVI Measures tested. Compass Diagnostic Tests in Arithmetic, Form A.

inch feet yard rod mile	ounce pound ton	peck bushel	pint quart gallon	second minute day hour year

Table LVII Types of reduction. Compass Diagnostic Tests in Arithmetic, Form A.

3 feet 2 inches	= ---- feet	
27 inches	= ---- yard	
7 2/3 yard	= ---- 7 yard --- feet	
1/4 peck	= ---- quart	
3 feet 15 inches	= ----	
6 hours 128 minutes 156 seconds	= ----	
8 Ton 1025 pounds	= ---- 7 Ton --- pounds	

3 feet 2 inches is to be written as 3 feet and what fraction of foot; 27 inches is what fractional part of a yard, and so on.

The examples in addition of denominate numbers range from two to five addends with two or three place figures.

The following are typical examples:

Add:

6 yard 1 foot 11 inches	6 bushel
7 yard 0 feet 2 inches	2 bushel 3 peck
6 yard 2 feet 1 inch	2 peck
7 yard 1 foot 0 inches	4 bushel 1 peck
6 yard 1 foot 10inches	5 bushel 2 peck
	3 bushel 3 peck

Leave each answer in reduced form. In subtraction; the examples range from one to three place figures in the minuend, and one to three in the subtrahend.

The following are typical examples:

9 gallon 2 quart	4 bushel 1 peck 3 quart
- 3 quart	- 2 bushel 3 peck 7 quart

In Tests XII which involves multiplication and division of denominate numbers, the work in reduction ascending and descending is carried still further.

The following are typical examples:

6 quarts 1 pint	4 pounds 11 ounces
x 8	x 9

answer in quarts and pints, reduce answer.

9)10 Ton 2500 pounds   10) 6 years 8 months

In Test XIII, Part 1, the vocabulary of mensuration is tested by means of recognition of figures and their various parts.

Example:

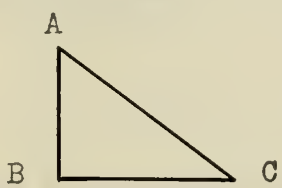

Figure 5

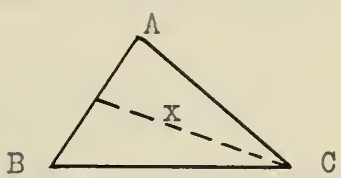

Figure 6

Figure 5 is a ------.

In figure 5, the line AC is called the ----.

In figure 6, the line x is an ---- and the line AC is a -----.

In Part 2, the ability to find areas of squares, rectanfles, parallelograms, and trapezoids is tested.

In Part 3, tests the ability to find volumes of rectangular solids, and prisms; Part 4, areas and volumes of circles, cylinders, and cones. In Part 5

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1. Sala, V., Denominate Numbers Used in the Factories of New Britain, Connecticut, Boston University, School of Education, Unpublished Thesis, 1931

formulas for the following were required:

Area of a rectangle	Area of a paralellogram
Area of a right triangle	Area of a circle
Circumference of a circle	Area of any triangle
Area of a trapezoid	Volume of a cube
Hypotenuse of a right triangle	Volume of an oblique prism
Volume of a cylinder	Volume of a cone
Volume of a pyramid	Sector
Area of the convex surface of a cylinder	Area of the convex surface of a cone

Test XIV Stresses the facts of percentage.

Table LVIII Equivalent Fraction; Decimal and percent, Relations, Compass Diagnostic Tests in Arithmetic, Form A.

Change Fractions to Percents	Change Percents to Fractions	Change Percents to Decimals	Change Decimals to Percents
9/100	6%	3%	.01
1/8	11%	4%	.0325
17/100	16 2/3 %	5%	.0475
1/4	20%	5 1/4%	.06
3/10	26%	6 3/4%	.065
1/3	30%	7 1/2%	.24
2/5	37 1/2%	12 1/2%	.4
4/5	62 1/2%	25%	.55
5/6	66 2/3%	33 1/3%	.7
7/8	70%	75%	1.00
9/9	75%	80%	1.13
3/2	80%	83 1/3%	1.63
1 3/4	100%	106%	
	150%	135%	
	175%	150%	

The three cases in percentage are tested. Five

solutions are given for each of twenty - five examples. The pupil is to choose the correct solution. Twenty eight other examples are given for the pupil to solve. The following illustrate the types of examples given.

1. Find 4 1/2% of 258.
2. Find 118% of 465.
3. 7905 is what percent of 6375?
4. If 150.64 is 28% of N, what is 100%?
5. If 6 1/2% of N equals 70, what is 100%? (Carry answer to two decimal places.)

The vocabulary of interest and business forms are tested in Part I, Test XV. It is a multiple choice test.

Example:

Underline the best answer.

In order to cash a check it must be ---- discounted; endorsed; receipted; balanced; sold.

Part II consists of filling in checks, stubs of checks, and itemized accounts. Parts III and IV contain examples and problems in finding interest by formula, and by means of interest tables.

The tests in problem analysis endeavor to test ability to read problems, understand them, estimate the answer, and determine the correct

solution. There are two forms, Form A, Elementary and Form B, Advanced. They are alternate response tests, five statements are given under each question. The following questions are asked for each problem.

1. Check true statement.
2. Check what is given.
3. Check what is called for.
4. Check probable answer.
5. Check correct solution.

Table LIX Situation Involved in Problem. Compass Diagnostic Tests. Problem Analysis: Elementary: Form A.

	Topic or Situation Involved in Problem
1.	Number of games won by baseball team
2.	Dividing books among children
3.	Number of sheep farmer sold
4.	Number of quarts of milk bought in one week
5.	Amount of money left when purchases were completed
6.	Boy saving money
7.	Number of miles travelled in one day
8.	Number of eggs found
9.	Spending money for a boy's baseball club equipment
10.	Sharing money earned
11.	Average price received for each fish
12.	Amount of change received
13.	Spending money buying horses and cattle
14.	Shopping for mother
15.	Earning and spending money

Table LX Situation Involved in Problem. Compass Diagnostic Tests. Problem Analysis: Form A.

	Topic or Situation Involved in Problem
1.	Giving away fractional part of apple
2.	Total amount earned by boy
3.	Number of miles a boy scout hiked
4.	Amount of coal left in coal bin
5.	Cost of curbing and paving to add price of each lot
6.	Time girl had to wait until train was due
7.	Amount grocer paid for melons and apples
8.	Profit on each chick
9.	Number of tomato plants which can be set out in garden
10.	Number of papers sold
11.	Number of quarts of varnish needed
12.	Amount agent should remit to owner
13.	Amount of money boy saved who did the work
14.	Rate per day workmen were paid for their labor
15.	Height to which second bin was filled when levelled

Tests XX and XXI of the Compass Diagnostic Tests are general problem scales.

The problems are similar to those contained in the Problem Analysis Tests but they are arranged in order of difficulty.

Table LXI Facts Used in the Fundamental Processes Compass Diagnostic Tests, Form A.

Primary Addition Facts	Upper Decade Facts	Higher Decade Facts	Primary Subtraction Facts	Multiplication Facts	Short Division Facts
79	81	2	58	77	49

Theoretically an inventory test covers completely the useful facts, primary and related in the fundamental processes, and deals with each significant type of difficulty in each operation separately.

The data of Table LXI shows that the Compass Diagnostic Tests do not ~~tests~~ test adequately the facts in the fundamental processes. Many skills are analyzed but few combinations are tested.

79 of the 100 primary addition facts, 21 of the 300 upper decade facts, and 2 of the 80 higher decade facts used in carrying in multiplication are tested; 58 of the 100 primary subtraction facts, 77 of the 100 multiplication facts, and 49 of the short division facts are tested. The Compass Diagnostic Tests afford a sampling of the useful facts, primary and related in the fundamental processes, and therefore, should be classified as a survey test.

The Compass Diagnostic Tests provide analysis of many skills in fraction work, but according to research work in the social usage[1] of arithmetic our actual need in fraction work is confined to such simple problems it is unnecessry to demand a high standard in manipulating artificial problems.

The following groupings of fractions are found only in the classroom; not in life outside.

Addition of fifths, sevenths, and thirty fifths; of halves, fifths, and sixths; of halves, sevenths, and fourteenths.

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1. Fourth Yearbook, Department of Superintendence, National Education Association, Chapter VII, Arithmetic, page 177

Subtraction of sevenths from halves; of fifths from halves; of fourteenths from halves; of twelfths from ninths.

Multiplication of twenty-fifths by sixteenths; fifths by fourteenths; twelfths by nineteenths; thirds by halves and by elevenths.

Division of twentieths by eighths; thirds by halves; sevenths by fourths; twelfths by twenty-fifths; sixteenths by fortieths. There is little or no need for division of fractions.[1]

In social usage decimals are used mainly with United States money[2]. It would be difficult to find situations in which the following decimals would be used by the average person.

1. 244 + 24.4 ÷ .244 + .0244
2. .0004 - .00019
3. .9182 x .2104
4. .452 87.5076

The work in addition, subtraction, multiplication,

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1. Upton, Clifford, Changing the Curriculum in Arithmetic, Teachers' College, New York, December, 1926, Record 28:341 - 559.

2. Woody, Clifford, Types of Arithmetic Needed in Certain Types of Salesmanship, Elementary School Journal, Volume 32, pages 505 - 520.

division of denominate numbers, involves obsolete practices[1]. Such examples as the following are found only in the classroom.

Add

```
 6 yard 1 foot 11 inches
 7 yard 0 feet  2 inches
 6 yard 2 feet  1 inch
 7 yard 1 foot  0 inches
 6 yard 1 foot 10 inches
```

Subtract

```
 8 Tons 1025 pounds
 -      1500 pounds
```

Multiplication

```
 3 feet 9 inches
          x 17
```

Divide

```
 6) 4 pounds 14 ounces
```

Work in mensuration should be informational[2].

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1. Sala, V. Denominate Numbers Used in the Factories of New Britain, Connecticut, Boston University School of Education, Unpublished Thesis. 1931

2. Fourth Yearbook, Department of Superintendence, National Education Association. Chapter VII, Arithmetic, page 177

The aim of informational subject matter is defeated when subjected to formal drill and testing. Drill should be kept behind meaning; therefore, formal definitions and rules of arithmetic are not memorized. The problems are the typical textbook problems. They present farfetched situations, or deal with material with which the child is not familiar. The following are examples.

1. Two weeks ago I had 6.7 tons of coal in my coal bin. The first week I burned 1.1 tons. The second week I burned only .8 tons. How much coal have I left?

2. At harvest time I had 956 bushels of wheat. I had one bin 11 feet 6 inches long, 6 feet wide, and 8 feet 4 inches deep which held 460 bushels when filled. I stored the rest of it in a bin 10 feet long and 7 3/4 feet wide. How high did it come in this bin when levelled off?

If written problems are developed out of the life experiences and activities of the pupils, most of the problem difficulties would disappear.

The aim of problem work is to develop understanding of business and develop judgment in the use of money. The pupils cannot develop judgment when presented with situations which they fail to understand.

The Compass Diagnostic Tests afford only a sampling of facts in the fundamental processes and should, therefore, be classed as a survey test. Tests I to IV deal with the fundamental processes and may be used for general survey purposes. Tests V to XX are not in harmony with nor do they reinforce the right curricular principles.[1] They test useless and traditional material. Standardized tests set up in the minds of teachers standards for the selection of material and instruction. They ought to represent in some degree forward looking tendencies in curriculum development.[2] As a whole the Compass Diagnostic Tests in Arithmetic are of doubtful value to the classroom teacher.

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1. Wilson, G.M., and Hoke, K.J., How to Measure, The Macmillan Company, New York, 1929, page 279

2. Simonds, P.M., Measurement in Secondary Education, The Macmillan Company, New York, 1930, page 280

Scientific studies on problem solving agree that most of the problem difficulties have been due to the fact that pupils did not have the experience necessary for understanding the situations presented in the textbook problems.[1]

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1. Fourth Yearbook, Department of Superintendence, National Education Association, Chapter VII, Arithmetic, page 177

d. Scale 1 of the Monroe Standardized General Survey Arithmetic Scales[1] is designed for Grades 3, 4, and 5. It consists of 8 tests. Scale II designed for Grades 6, 7, and 8 consists of 7 tests. In Test 7, the pupils are asked to insert decimal points in quotients; in all others they are asked to do arithmetic examples. There are three forms for each scale. All three forms are similar in content.

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1. Monroe, W.S., Standardized General Survey Arithmetic Scales, Public School Publishing Company, Bloomington, Illinois

The following process steps are used in the Monroe Standardized General Arithmetic Scales.

Addition

1. Primary facts.
2. Decade drill.
3. Single column addition, five addends, without zeros.
4. Single column addition, with zeros.
5. Four place figures, five addends, without zeros.
6. Four place figures, five addends, with zeros.

Subtraction

1. Primary facts.
2. Related facts.

   Three or four place figures in the minuend.
3. Gaps, no borrowing.
4. Gaps, borrowing.

Multiplication

1. Primary facts.
2. One place multipliers, carrying, requiring addition in higher decade.
3. One place multiplier, zero in multiplicand, with and without carrying in zero.

4. Two or three place multipliers, with carrying.
5. Two or three place multipliers, carrying, single zero in multiplier or multiplicand.

Short Division

1. Primary facts.
2. Two or more digits in quotient, carrying, no remainder.
3. Zero in quotient with or without carrying, no remainder.

Long Division

1. Involving chiefly the form of operation in long division; divisor of two places and its right hand figure very small, no carrying in multiplication; no borrowing in subtraction; no remainders.
2. Two place divisor, four or five place dividend, carrying, borrowing, but no quotient difficulty.
3. Quotient difficulty
   Trial divisor is not the correct quotient.

Table LXII Addition facts and their frequency. Monroe's Standardized General Survey Arithmetic Tests. Scale 1, Form 1.

	0	1	2	3	4	5	6	7	8	9
0	1	2		1	1		1		1	1
1	1		1	3	3	3		1	1	1
2		1	1	1	2	2	1	1		
3		1		1	1	1	1		2	2
4	2	1	2	3		2		1	1	1
5	1	1	3	3	1	3	1	1		1
6	2	2	2		1	1	1	3	3	3
7	2		2	2	1	2	1	2		1
8	1	1	1	3	2	2		2	1	2
9		1	2		2	1	1	2	1	2
10			2	1					2	
11					1		1	1	1	
12		1		1	1	2	1	1	1	1
13	2	1			2	2		1	2	
14							1		1	
15		1	3							
16			1		1		1			1
17		1		1			1	1	1	1
18		1	1	1	2	1		1	1	
20		1		2						
21							1			2
22			1				1			
23							1			
24				1						
25							1			
26						1				1
27		1								

Table LXIII Addition facts and their frequency. Monroe's Standardized General Survey Arithmetic Tests. Scale 1, Form 1.

	0	1	2	3	4	5	6	7	8	9
1	1		2	2	1	1	1	1		
2	3	3	4	3	2	2	1	4	2	4
3		2	2	3	4		2	1		2
4			1	2		2	1	2	1	2
5		1	1				1	1		
6		1	2			2	1		2	3
7	1	1	2	1	1	5		2	1	1
8			1		1	1			2	1
9	3	1	1		2	2	1		1	1
10			1	1				1	3	
11	2	3		1		2	3	1		2
12	4	1	3	3	1	1		5	1	2
13	2	2	1	1		2		1		
14		1	1	1	2		2		2	1
15			1	1	1			1	1	1
16	3	1	1	1	2	1		2		1
17		1		1	2	1	2	2		
18	1	1	2		2	1	1	2	3	1
19				1	2			1		2
20			1						3	1
21					1	1		1	1	
22	2	2	1			2			1	
23		1	1		1		2	2		1
24	1								1	
25								1		
26				1	1			1		1
27							1			
28			2						2	
29							1			
33								1		

The numbers at the left are added to those across the top of the table. The number in the square denotes the frequency of the combination.
80 of the primary addition facts and 54 upper decade facts are used in Scale 1, Form 1; 63 of the primary addition facts, and 96 upper decade facts are used in Scale II, Form I.

Table LXIV Addition facts and their frequency. Monroe Standardized General Survey Arithmetic Tests. Scale 1, Form II.

	0	1	2	3	4	5	6	7	8	9
0		1	1	1	1					
1		1		3	4	3	1	1		
2	1		1	1	1	2	1			1
3		1	1		1	2	1	1	2	1
4	1	1	2	3	1	2		1	2	2
5			2	3	1	3	1	3	1	
6	1	1	3	1	1	2	2	3	2	2
7	2	2	2	2	1	2	1	2		
8		2	2	2	1	1	1	1	2	3
9	2	1	2	1	2		1	2	1	
10							1		4	1
11		1					2	1	1	
12		2		1	1				1	1
13	1		1	2	1	1	1			1
14				2	1					
15	1		1		1		1			
16		1			2			1	1	
17		1	2		1	2	1	1		1
18						2		2		1
19			2		1		1			
20			1			1				1
21				1						
22						1			1	
23						1		1		
24		1					1			
25			1	1						

Table LXV Addition facts and their frequency. Monroe Standardized General Survey Arithmetic Tests, Scale II, Form II.

	0	1	2	3	4	5	6	7	8	9
1		1	2		2		1	2	1	3
2	3	1	2	3	3	2	4	6	2	3
3		1	3	4	2	1	2	2		
4						1				3
5			2	2	2	1		1		1
6	1		1		1		1	3	1	2
7			2	1		2			1	1
8		1	1			6	1		2	
9	2				1	1	1	2	2	3
10	1		2	1	2	1		2	2	2
11	1		1		1					2
12	2	2	1		1			1		1
13	2	3	1	2		2	2	2	2	1
14	1		1	3	1	1	3	3		
15	2	2		1	1	3		2		
16	2	1		1	3		1	3		
17	1	1	1			1		2	1	1
18	1		1	2		1		2	1	2
19		1	2	1	1				1	
20		1	1		3	1	1		4	
21		2	1				2		1	2
22	1		1		1			1		1
23				1				2		
24	1					1		2	1	
25			1						1	
27			1		1		1			
28			1		1					1
29							1		1	
33									1	

The numbers at the left are added to those across the top of the table. The number in the square denotes the frequency of the combination.
76 of the primary addition facts and 54 upper decade facts are used in Scale 1, Form II; 56 primary facts and 96 upper decade facts are used in Scale II, Form II.

Table LXVI Addition facts and their frequency. Monroe Standardized General Survey Arithmetic Tests, Scale 1, Form III.

	0	1	2	3	4	5	6	7	8	9
0	1	1				1	1	1	1	
1	2		1	2	4	2	2			
2		1	1		1	1		2	3	2
3	1	1	1	1	1	1	1	1	1	
4	2		2	1		1	2	2	2	2
5		2	2		1	1		1	1	2
6			3	2		2	2	1	1	1
7		1	2	1	1	1	1	1	1	2
8	1	1	1	2		1	2			1
9	1	1		3	1	1	1	2	3	1
10		1	1	1		2			1	1
11	1				2	3				1
12				1	2	1		1		2
13		1	1	1					2	
14				1	2		1			
15	1	1	1	2		1	1			
16		4	1		1	1		1		
17	2					3	1	1	1	
18		1	1		1	1				
19			1	1					1	
20		1		2						
21						1	1	1	1	
22		1		1	1					
23		1				1			1	
24							1			
26						1				1
27				1						

Table LXVII Addition facts and their frequency. Monroe Standardized General Survey Arithmetic Tests, Scale II, Form III.

	0	1	2	3	4	5	6	7	8	9
1	1		3	3	3	1	2	2		
2		3	4	2	1		3	6	2	3
3	2	1	3	2	3		1	3	1	3
4	1	1	1	1		1	1	1	1	3
5				1	1	4			3	1
6	1		2			1	1	2		
7	2	1			1	3		1		1
8	1		1	1		2		2		2
9	1				3	3	2		1	1
10	1	2	1	1	2		1		3	1
11	2	2	1	1	1	3	1	1		1
12		1	2	2				1	1	2
13	4	2	1	1	1	3	1	3		2
14	2		1		2	3	2	1	1	2
15	2	2	1	3						
16		1		1	1			2	2	
17				1	2		2	2		
18	1	1	1.	1				2	2	1
19	2	2	1					3	2	
20					1		1	1	3	3
21			1		1			1		
22								2		
23			2	1			1			
24		1	2	1						
25						1				
26					2	1	1		2	
27			1		1					1
28								1	1	1
29									1	1
30							1			
31			1							

The numbers at the left are added to those across the top of the table.
In Scale 1, Form III, 75 primary addition facts and 61 upper decade facts are used; in Scale 2, Form 3, 61 of the primary addition facts and 95 upper decade facts.

Table LXVIII Subtraction facts and their frequency. Monroe Standardized General Survey Arithmetic Tests, Scale I, Form I.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1				1	1	1	1										
1	1	1			1	1		1	1	1								
2			1	1	1			1	1	1								
3				1	2	1	2	1		2		1						
4				1	1	1			1	1	2	1	1					
5					1		2	1		2	1	1	1					
6							1	1		1	1	1		1	1			
7									2	2	1	1	2	1		1		
8											2		2	2	2	1		
9									1	1	1	1	1		1	1	2	1

The numbers at the left of the table are subtracted from those across the top. The borrowing method of subtraction was used. The number in the square denotes the frequency.
68 subtraction facts are used in Scale 1, Form 1.

Table LXIX Subtraction facts and their frequency. Monroe Standardized General Survey Arithmetic Tests, Scale 1, Form II.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	1	2				1		1	2									
1		3		1	1					1								
2			2	1	1	3	1	2		2	2							
3						1	3	1				2	1					
4					3	1		1	1				1					
5						2	1		1	1	3	1	2					
6							1	1	1	1	2	1		2	1			
7									1		1			1	1	1		
8									1	1	2	1	1	1		1	1	1
9												3		2		1		1

The numbers at the left of the table are subtracted from those across the top. The number in the square denotes the frequency.
54 subtraction facts are used in Scale I, Form 2.

Table LXX Subtraction facts and their frequency. Monroe Standardized General Survey Arithmetic Tests. Scale 1, Form III.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	1	3	2			1		1		1							
1		3		1		1	1	1	1								
2					1	1	1		2	1			1				
3				1	1			1		1	3		2				
4					1	2	1	3	1	1	2			2			
5							1	1	1		1	1		1			
6							1			1	1		2		1		
7									1		1	2	1	2		2	2
8										1	2	1	1	2	2	1	1
9													1				4

The numbers on the left are subtracted from those across the top of the table.
60 subtraction facts are used.

Table LXXI Subtraction facts and their frequency. Monroe Standardized General Survey Arithmetic Tests. Scale II, Form III.

	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	1			3										
1		2		2	1									
2		1	2	3	1			1						
3				2	3			3						
4		2	3		2		1							
5				1		2	1	1		1				
6				2	1		1		2	1	2			
7					1	2	2		2	1				
8						1	3		2	2		1	1	
9						2		1	1		1			1

The numbers on the left of the table are subtracted from those across the top.
44 subtraction facts are used.

Table LXXII Subtraction facts and their frequency.
Monroe Standardized General Survey Arithmetic Tests.
Scale II, Form I.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	1				1	1											
1						2	2			1								
2			1	1	2			3		1								
3				1		1	1	2	1	1	1							
4				2		2	1	1	1	1								
5							1	1	1		1	1	1					
6							1	2		1			2	1	2			
7								1	2	1	1	3				1		
8										2		2	1		2		2	
9									1	1		1			1	1		1

The numbers at the left of the table are subtracted from those across the top. The number in the square denotes the frequency.
60 subtraction facts are used in Scale 2, Form 1.

Table LXXIII Subtraction facts and their frequency.
Monroe Standardized General Survey Arithmetic Tests.
Scale II, Form II.

	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	1		1			1									
1			1		2	1		1								
2			1	1	1	2	1	2								
3	1		1		2			2		2						
4		1	1	1	2		1	1			1					
5			1		2	1	1		1							
6					2		2		2		3					
7					1	2		2	1	1			1			
8							1		1		3		2		3	
9									2				1	1	1	1

The numbers at the left of the table are subtracted from those across the top. The numbers in the squares denote the frequency.
51 subtraction facts are used in Scale 2, Form 2.

Table LXIV Multiplication facts and their frequency. Monroe Standardized General Survey Tests, Scale I, Form I.

	0	1	2	3	4	5	6	7	8	9
0	1		1		1			1		
1			1		1			1		1
2	2	1	2	2	1	3	2	1	2	1
3	1		1	2	2	1	2	1	2	2
4		1	1	2	2	1	1	2	1	1
5	2	2	1	1	2	2	2	2	1	3
6	2	2	1	1	3	2	1	3	1	2
7	1	1	1	1	2	1	1	2	1	1
8			2	2	2	1	2	1	2	2
9	1	1	2	2	1	1	2	3	2	2

The numbers on the left are the multipliers.
84 multiplication facts are used in Scale 1, Form 1.

Table LXV Multiplication facts and their frequency. Monroe Standardized General Survey Tests, Scale II, Form I.

	0	1	2	3	4	5	6	7	8	9
2		1	3		2	2	3	1	3	1
3		1	3	1	2	4	1	1	1	2
4	1	1	1	3	1	2	3	2	1	1
5			2	1	3	2	2	2	1	3
6		1	1	3	1	3	2	3	1	1
7	1		1	1	3	2	4	2	2	
8	1	1	2	3	1	2	1	2		3
9	1	1	2	1	1	1	2	3	3	1

The numbers on the left are the multipliers.
71 multiplication facts are used in Scale 2, Form 1.

Table LXVI Multiplication facts and their frequency. Monroe Standardized General Survey Tests, Scale I, Form II.

	0	1	2	3	4	5	6	7	8	9
0	1	1							1	1
1	1	1		1	1				1	
2		1	2	2	3	2	1	3	2	1
3		1	1	1	2	3	1	1	3	
4	1	1	1		1	1	1	2		2
5	1	1		2	3	1	2	3	3	1
6	1		1	2	3	1	3	1	2	3
7	1	1	2	1		2	2	3	1	1
8	1	2	1	1	2	1	2	3	1	1
9	1	1	2	2	1	2	1	3	2	3

The numbers on the left are the multipliers.
81 multiplication facts are used in Scale 1, Form 2.

Table LXVII Multiplication facts and their frequency.
Monroe Standardized General Survey Tests, Scale II,
Form II.

	0	1	2	3	4	5	6	7	8	9
2		2	3	2	2	3	1	1	1	1
3			3		3	2	3	1	3	1
4	1	1	1	2		2	1	4	2	2
5			2	1	3	1	4	3	2	1
6		1	1	2		3	2	3	2	2
7	1		3	1	2	2	1	2		4
8	1		1	1	3	2	4	2	2	
9	1	2	2	3	1	3	2	1		1

The numbers at the left are the multipliers.
66 of the multiplication facts are used in Scale II,
Form II.

Table LXVIII Multiplication facts and their frequency.
Monroe Standardized General Survey Tests. Scale I Form III.

	0	1	2	3	4	5	6	7	8	9
0	1		1	1		1	1	1		
1					1	1	1	1		
2	1	1	3	1	1	2	1	2	3	2
3			2	2	3	1	1		3	1
4	2	2	1	1	1	2	2	3	1	1
5	1	1	1	1	3	2	2	1	2	1
6	1		1	3	2	2	2	1	2	2
7	2	2	1		1	1	2	3		2
8		1	1	2	1		1	1	2	2
9		1	2	3	1	2	2	2	2	3

The numbers at the left are the multipliers.
81 of the multiplication facts are used in Scale I,
Form III.

Table LXIX Multiplication facts and their frequency.
Monroe Standardized General Survey Tests. Scale II,
Form III.

	0	1	2	3	4	5	6	7	8	9
2		1	1	3	1	3	1	3	1	2
3		2	1	3		2	3	3	2	
4			3		3	2	3		1	3
5	2		2	1	2	1	3	2	2	1
6		2	3	1	1	3	2	1	2	1
7	1	1	3	2	1	2	1	2	1	2
8	1		1	1	3	2	4	2	2	
9			1	1	3	3	1	3	2	2

The numbers on the left are the multipliers.
66 of the multiplication facts are used in Scale II, Form III.

Table LXX Short Division facts and their frequency. Monroe Standardized General Survey Tests. Scale I, Form I.

	1	2	3	4	5	6	7	8	9
0	1	1	2		2	3	1	1	
1	2								
2	1								
3	1	1	1						
4		1		1					
5			1						
6		2	2	1		2			
7	1			1		2	1		
8	1	1	2						
9			1		1		1		1
10		2		1	2				
11				1		1			
12						1			
13				1					
14							1		
15			1						
16				1					
17		1							
18		1				1			
20				2					
21			3				1		1
22				1					
23				1			1		
24			2	1				1	

	1	2	3	4	5	6	7	8	9
25							1		
26			1				1		
27			2						1
28							2		
30					1		1		
31					1			1	
32				2				1	
34				1					1
35					1				
36						1			1
38				1				1	
42						2	1		
43					2		1		
47									1
48						3			
49							1		
53									1
58						1			
63							2		2
64								2	
72								3	1
81									1
85									1
88									1

The figures at the top are the divisors. 90 of the short division facts are used in Scale 1, Form 1.

Table LXXI Short division facts and their frequency. Monroe Standardized General Survey Tests. Scale I, Form II.

	1	2	3	4	5	6	7	8	9
0	1	1	2			3	1	1	2
2		1							
3	1	1							
4		2							
5	1	1	2						
6		2	1			3			
7				1		1			
8			1	1					
9	1				1		1		
10		3			1				
11		2				1			
12		1	1			1			
14							2		
15		1			1				
16				1			1	1	
18		1	2	1					
19				1					
20				1					1
21			2						
23								1	
24			1	1			1		
25			1		2		1		
26					1				
27			1	1					3
28				1			2		

	1	2	3	4	5	6	7	8	9
29							1		
30						1			
32				2					
33				1	1				
35					2				
36						1	1		1
37				1				1	
38									1
40								1	
42					1	3	1		
43					1				
45					1				1
46						1			
48							1		
49							1		
52						1			1
54									1
56							1	1	
63							2		1
64								1	
72								3	
78									1
81									1
84									1

The figures at the top are the divisors. 92 of the short divisor facts are used.

Table LXXII Short division facts and their frequency.
Monroe Standardized General Survey Arithmetic Tests
Scale I Form III

	1	2	3	4	5	6	7	8	9
0	1		2		1	3		1	
1	1								
2	1								
3		1	2						
4	1	1		1					
5	1	1	2		1				
6		1	4			3			
7				1		2			
8			1	1					
9	1		1		1		1		
10		3			1				
11		2				1			
12						1			
14	1	1							
15					1				
16		1							
18		1	1			1			
20				1	2				
21			3				2		1
22				1			1		
23						1		1	
24			2				1		
25							1		
26			1	1					
27			1						1

	1	2	3	4	5	6	7	8	9
28				1					
30					1		1		
32				1				2	
34				1					
35								1	
36				1	1				2
38				1	1				
42						1			
43					1				
44							1		
46						2			
48						1		1	1
49							1		
52						1			
54						1	1		1
56							1		
57							1		
63									1
64								1	
68								1	
72									1
74								1	
81									1
85									1
86									1

The figures at the top are the divisors.
89 of the short division facts are used in Scale I, Form 3.

Table LXXIII Divisors and the number of digits in dividends Scale II Form I, Monroe Standardized General Survey Tests, Scale II, Form I.

	21	23	27	31	32	41	42	47	51	52	59	67	68	72	76	78	84	89	96	98
3	1	1		1		1	1		1	1										
4					1									1			1			
5			1					1			1	1	1		1	1		1	1	1

Table LXXIV Divisors and the number of digits in dividends. Monroe Standardized General Survey Tests Scale II, Form II.

	21	23	28	31	32	41	42	48	51	52	58	67	68	73	76	79	84	89	96	98
3	1	1			1	1	1		1	1										
4				1										1			1			
5			1					1			1	1	1		1	1		1	1	1

Table LXXV Divisors and the number of digits in dividends. Monroe Standards General Survey Tests Scale II, Form III

	21	24	29	31	32	34	38	41	42	46	49	51	52	56	67	73	79	84	89	98
3	1				1															
4				1				1	1	1		1	1			1		1		
5		1	1			1	1				1			1	1		1		1	1

The numbers across the top of the table are divisors, those at the left are the number of digits contained in the dividends.

Table LXXVI Number of Facts Used in the Monroe Standardized General Survey Tests.

Scales Forms	100 Primary Addition Facts	300 Upper Deacde Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
Scale 1 Form 1	80	54	0	68	84	90
Scale 2 Form 1	63	96	0	54	71	
Scale 1 Form 2	76	54	0	60	81	92
Scale 2 Form 2	56	98	0	51	66	
Scale 1 Form 3	75	61	0	60	81	89
Scale 2 Form 3	61	95	0	14	66	

The data of Table LXXVI shows that for a survey test The Monroe Standardized General Survey Tests give a sampling of the facts in the fundamental processes. They are similar to the Cleveland Survey and the Compass Survey Tests. In division of fractions the following groupings were noted: sevenths and fifths, sevenths and fourths; fifteenths and eighths, elevenths and sevenths. These fractions are still found in

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1. Dalrymple, C.O., Frequency of Occurrence of Fractions in Industry, Boston University School of Education, Unpublished Master's Thesis, 1932

[illegible]

[illegible]

[illegible]

some arithmetic books, but according to recent studies in fractions[1] division of fractions and the above groupings of fractions are not used in actual business practice, and are, therefore, of doubtful value in the classroom. The tests in division of decimals are of doubtful value, also. According to newer courses of study in arithmetic it is recommended that the divisors be whole numbers,[2] preferably confined to United States money. By omitting the tests in fractions and decimals, the Monroe Standardized General Survey Tests may be profitably used for general survey purposes.

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1. Dalrymple, C.O., Frequency of Occurrence of Fractions in Industry, Boston University School of Education, Unpublished Thesis, 1932

2. Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin, Company: Boston, Massachusetts, Chapter XIV

e. The Monroe Diagnostic Tests in Arithmetic do not meet the standards of inventory tests and therefore, do not come under that classification.

The Monroe Diagnostic Tests[1] are composed of a series of twenty-one tests, requiring thirty-one minutes to administer, and are designed to diagnose the particular difficulties of each individual child. These tests are printed in four parts. Standard scores are given. Rate in the Monroe Diagnostic Tests indicates the number of examples attempted, and accuracy is the percent of examples right. The scores on the individual tests are designed to enable the teacher to determine on what phases of the subject the pupil may be deficient, in so far as the tests contain examples dealing with all phases of the process.

Tests I to XI of the Monroe Diagnostic Tests in Arithmetic consist of examples on integers and are designed for Grades IV to VIII. Part III is on common fractions and is designed for Grades V to

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1. Monroe, W.S., Diagnostic Tests in Arithmetic, Public School Publishing Company, Bloomington, Illinois

VIII; Tests XVIII to XXI (Part IV) consist of examples in multiplication, and division of decimals. They are to be used in Grades VI to VIII. Each test is timed separately.

The following process steps are used in the Monroe Diagnostic Tests.

Addition

1. Single column addition, no zeros, sums to 20.
2. Single column addition, with zeros, sums to 20.
3. Single column addition, thirteen addends, with and without zeros.
4. Column addition, four place figures, five addends, without zeros.
5. Column addition, four place figures, five addends, with zeros.

Subtraction

1. Related facts.
2. Simple subtraction with one step borrowing.
3. One step borrowing with occassional vanishing lefts.
4. Double borrowing, gaps.
5. Triple borrowing.

Multiplication

1. One place multiplier, carrying.
2. One place multiplier, carrying, single zero in multiplicand.
3. Two or three place multipliers without zeros.
4. Two or three place multipliers, zero in multiplicand with carrying in zero.
5. Zero in multiplier and in multiplicand.

Short Division

1. Two or more digits in quotient, carrying, no remainder.

Long Division

1. Involving chiefly the form of operation in long division; divisor of two places, and its right hand figure very small, no carrying in multiplication, no borrowing in subtraction; no remainders.
2. Two place divisor, three or four place dividend, carrying, no borrowing, no remainder.
3. Two place divisor, three or four place dividend, borrowing, no remainder.
4. Quotient difficulty

   Trial divisor not the correct quotient.

Table LXXVII Addition facts and their frequency Monroe Diagnostic Tests, Part 1

	0	1	2	3	4	5	6	7	8	9
0					1			2		
1	1		3	2	1	1	2	1		1
2	2	3	3	3	2	2	1	5	2	5
3		4	4	2	4		2	1	1	3
4		2	1			2	1	2	2	2
5	2	2	2		1		2	3	1	
6		1	1	1		2	1		2	1
7	1	1	3	1	3	6	2	3	1	2
8	1		1	1		2			2	1
9	5	1	1	2	3	3	1	3	2	3
10			1	1		1		1	3	1
11	3	3	1			2	4	1	1	2
12	4	2	3	3	1	1	1	5	1	2
13	2	2	2	1	1	2		1		
14		1	1	1	2	1	2		2	1
15			1	1	1			1	1	1
16	3	1	1	2	3	1		2		1
17		1		1	1	1	2	2		
18	1	1	2		1	1	1	2	3	1
19				1	3			1		2
20									3	
21					1	1		1	1	
22	2	2	2			1			1	
23		1	1		1	1	2	2		1
24	1								1	
25								1		
26				1	1			1	1	
27							1			
28			2						2	
29						1				
30										1
31										
32										
33								1		1
35										
36										
37										
38										

Table LXXVIII Addition facts and their frequency. Monroe Diagnostic Tests, Part 2

	0	1	2	3	4	5	6	7	8	9
0										
1										
2		3								
3			1	1						
4			2	1		1				
5		2				1				
6		2	1	1	1		1			
7	1		1	2			1	2		
8	2	1							1	
9		2			1	1		1		
10	1		2	1	1	1	1			
11										
12						1				1
13				1			1	1		1
14			1		1	1				
15										1
16					1	1	1		1	1
17							1	1		
18			1							
19	1				1				1	
20					2			1		
21									1	
22		1	1							
23				1					2	
24	1	1	1		1					1
25		1		1	1	2		1		
26				1			1	1		
27							1		1	
28		1		1						
29					2				1	
30	1	2				1		1		
31								1	2	
32		1			1					
33	1		1	1	1		2			1
35		1					1			
36	1			1		1	1			
37	1		2	1						
38		1								

Monroe Diagnostic Tests in Arithmetic
Continued

	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
39												1		2	1	1	1	1		1
40												1								1
41											1						1	1		1
42													1				1		2	
44													1						1	1
46													1			1				1
47																		1		
48															1		1	1		
49														1						
50												1		1						1
51														1						1
52															1			1		
53											1	1							1	1
55																			1	
56														1						
59																			2	1
60																			1	
61																		1		
62													1							
63																1				
67																			1	
85																	1			

The numbers on the left are added to those across the top of the table. The number in the square denotes the number of times each combination occurs.

In Part 1, 75 primary facts in addition, and 103 upper decade facts are used.

In Part II, 25 primary facts, 84 upper decade facts, 41 higher decade facts used in carrying in multiplication are used.

Table LXXIX Subtraction facts and their frequency. Monroe Diagnostic Tests, Part 1.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1							1											
2						1			1									
3						1		1			2							
4						1					1							
5								1					1					
6								1							1			
7										1	1			1				
8												1		1	2			
9														1			1	2

Table LXXX Subtraction facts and their frequency. Monroe Diagnostic Tests, Part 2.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0		1	1				1	1											
1							2	2			1								
2				1	1	2			3		1								
3					1		1	1	2	1	1	1							
4					2		2	1	1	1	1								
5								1	1	1		1	1	1					
6								1	2		1			2	1	2			
7									1	2	1	1	2				1		
8											2		3	1		2		2	
9										1	1		1			1	1		1

The numbers on the left of the table are subtracted from the numbers across the top of the graph. The number in the square denotes the frequency of occurrence.

21 primary subtraction facts are used in Part I and 54 in Part 2.

Table LXXXI Multiplication facts and their frequency.
Monroe Diagnostic Tests, Part 1.

	0	1	2	3	4	5	6	7	8	9
0	8	4	2	3	4	3	5	4	5	4
1		1						1	1	
2	4	1	1	4	1	3	2	3	4	1
3	4		2	3	4	4	3	1	1	4
4	3		2	2	1	2	3	3	3	1
5	4	2	1	2	1	1	2	6	3	2
6	5	1	3	3	2	2	1	5	2	3
7	3		1	2	1	3	4	1	3	2
8	5	2	2	2	5	4	3	2	2	2
9	5	2	2	2	1	2	3	5	4	1

Table LXXXII Multiplication facts and their frequency.
Monroe Diagnostic Tests, Part 2.

	0	1	2	3	4	5	6	7	8	9
2		1	1			2	1	1	1	1
3		1	3	1	2	4	1	1	1	2
4	1	1	1	2	1	1	3	1	1	
5			2		3	1	2	1	1	2
6		1	1	2	1	2	2	2	1	
7	1			1	2	1	2	1		
8	1	1	3	1	1	1	1	1		
9	1	1	1	1		1	1	3	2	1

The numbers at the left of the table are the multipliers.
90 of the multiplication facts are used in Part 1;
64 of the multiplication facts in Part 2.

Table LXXXIII Short division facts and their frequency. Monroe Diagnostic Tests, Part 1

	2	3	4	5	6	7	8	9
0		1			2			
3	1	1						
5	1	1						
6	1				1			
7			1		1			
8		1						
9				1		1		
10	2			1				
11	1							
12					1			
13	1							
14						1		
15	1							
20			1					
21		1						
22			1					
23			1					
24		1						
25						1		
26						1		
27		1						1
31				1			1	
32			1					
34			1					
38			1				1	
43				1		1		
46					1			
47								1
48					1			
53								1
55						1		
63						1		1
64							1	
72							1	
75							1	
81								1
87								1

The figures at the top are the divisors. 49 short division facts are used in Part 1.

Table LXXXIV Divisors and number of digits in dividends. Monroe Diagnostic Tests, Part 1.

	21	23	24	31	32	33	41	42	43	51	52	53	62	63	64	71
3		1						1								
4	1		1	1	1	1	1		1	1	1	1	1	1	1	1

	73	74	82	83	84	92	93	94
3								
4	1	1	1	1	1	1	1	1

Table LXXXV Divisors and number of digits in dividends. Monroe Diagnostic Tests, Part 2.

	28	36	47	48	57	67	68	76	79	89	96	98
5	1	1	1	1	1	1	1	1	1	1	1	1

The numbers at the left are the numbers of digits in the dividends, those at the top of the graph are the actual divisors.

Table LXXXVI Fractions used in addition. Monroe Diagnostic Tests.

1/2 + 2/3	3/5 + 1/2	3/8 + 5/6
1/2 + 7/10	4/5 + 7/10	5/8 + 1/4
1/2 + 7/12	1/6 + 1/3	5/8 + 3/4
1/3 + 3/4	1/6 + 2/3	4/9 + 1/6
1/3 + 4/7	1/6 + 3/5	5/9 + 2/3
1/3 + 1/12	5/6 + 1/2	1/10 + 1/15
1/4 + 5/8	5/6 + 7/12	3/10 + 1/4
3/4 + 1/2	1/7 + 2/5	3/10 + 2/5
3/4 + 5/12	4/7 + 3/5	7/10 + 3/8
2/5 + 2/3	1/8 + 1/2	4/15 + 5/9

Table LXXXVII Fractions used in subtraction. Monroe Diagnostic Tests.

1/2 - 2/7	3/4 - 2/7	5/6 - 2/15
2/3 - 1/2	4/5 - 1/3	7/9 - 1/6
2/3 - 3/5	5/6 - 3/4	7/10 - 1/6
3/4 - 1/3	5/6 - 3/5	7/12 - 3/8
3/4 - 2/5	5/6 - 3/8	3/15 - 4/9

Table LXXXVIII Fractions used in multiplication. Monroe Diagnostic Tests.

1/2 x 1/3	2/5 x 3/7	3/8 x 1/4
1/3 x 1/2	4/5 x 1/3	4/9 x 2/5
1/3 x 3/8	1/5 x 7/9	5/12 x 3/5
2/3 x 3/4	1/5 x 3/10	7/12 x 4/7
2/5 x 3/4	2/7 x 1/6	4/15 x 5/8

Table LXXXIX Fractions used in division. Monroe Diagnostic Tests.

1/2 ÷ 1/3	2/5 ÷ 3/7	4/7 ÷ 8/11
2/3 ÷ 3/4	3/5 ÷ 3/4	4/7 ÷ 2/3
2/3 ÷ 8/9	4/5 ÷ 1/2	3/8 ÷ 2/3
1/4 ÷ 1/6	5/6 ÷ 5/8	5/12 ÷ 4/9
2/5 ÷ 1/3	3/7 ÷ 4/5	7/12 ÷ 4/9

Table XC Value of last figure in divisor and dividend. Monroe Diagnostic Tests.

Example .415 ÷ .05, thousandths ÷ hundredths.

	Integers	Tenths	Hundredths	Thousandths
Integers	1	2	2	2
Tenths	8	8	5	9
Hundredths	2	10	9	10
Thousandths				
Ten-thousandths				

At the left of the table are the divisors.

Table XCI Value of last figure in multiplier and multiplicand. Monroe Diagnostic Tests.

	Integers	Tenths	Hundredths	Thousandths
Integers				
Tenths	3	6	7	6
Hundredths	2	8	5	7
Thousandths				

Tests XVII, XVIII, XIX, and XX comprise examples in multiplication, and division of decimals, in which the decimal point has to be inserted.

Table XCII Decimals used in division. Monroe Diagnostic Tests.

	Units	Tenths	Hundredths	Thousandths
Units		2	2	2
Tenths	9	8	6	8
Hundredths	1	9	10	10

The table is read as follows: tenths are divided by units, twice; hundredths are divided by units twice; thousandths are divided by units twice.

Table XCIII Decimals used in multiplication. Monroe Diagnostic Tests.

	Units	Tenths	Hundredths	Thousandths
Units		1		
Tenths	3	5	7	6
Hundredths	2	7	6	7

Table XCIII is read as follows:units are multiplied by tenths, three times; tenths are multiplied by tenths, five times; hundredths are multiplied by tenths, seven times; and so on.

Table XCIV Summary of Facts Used in the Monroe Diagnostic Tests.

100 Primary Addition Facts		300 Upper Decade Facts		80 Higher Decade Facts		100 Subtraction Facts		100 Multiplication Facts		Short Division Facts	
Part		Part		Part		Part		Part		Part	
1	2	1	2	1	2	1	2	1	2	1	2
75	25	103	84	0	41	21	54	90	64	49	

From the data of Table XCIV it may be seen that the Monroe Diagnostic Tests are diagnostic in name only. Only 75 of the primary addition facts out of the 100, 103 of the 300 upper decade facts, 21 of the 100 subtraction facts, 90 of the 100 multiplication facts, and 49 of the short division facts are used in Part 1. 25 of the primary addition facts, 84 of the 300 upper decade facts, 41 of the higher decade facts; 54 of the 100 subtraction facts; 64 of the multiplication facts are tested in Part II.

They do not afford a complete inventory of the useful facts, primary and related, in the fundamental processes. They afford a sampling of facts and, therefore, should be classed as a survey test.

The following appear to the author as pertinent criticisms.

Test VII, Part II, single column addition with thirteen addends, goes beyond social usage.[1]

Most of the fraction combinations are used only in textbooks of arithmetic and not in real life situations, for example, 4/7 + 3/5; 4/15 + 5/9; 7/9 - 1/6; 5/6 - 2/15; 7/12 x 4/7; 4/5 x 7/9. All the examples in division of fractions involve mere manipulation and disregard the principle of social usage.[2]

In business and social usage most of the work in decimals is in connection with United States money, therefore most of the work in decimals should be confined to United States money in the schools. Otherwise the work is meaningless involving mere manipulation. Such examples as the following do not appear to be based on life situations. They do not refer to United States money.

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1. Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin Company, Boston, Massachusetts, 1926, page 124.

2. Dalrymple, C.O., Frequency of Occurrence of Fractions in Industry, Boston University School of Education, Unpublished Master's Thesis, 1932

7.465 x 4.3

5.376 x .91

2.893 x .68

2758.9 ÷ .47

22572 ÷ .57

31.824 ÷ .68

By omitting Test VII, Part II, which goes beyond social usage, Parts I and II of the Monroe Diagnostic Tests may be used for general survey purposes. Parts III and IV are not in harmony with, nor do they reinforce the right curricular principles.[1] They test useless and traditional material in fractions and decimals.

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1. Wilson, G.M., and Hoke, K.J., How to Measure, The Macmillan Company, New York, 1929, page 279

f. The Pittsburgh Arithmetic Scales[1] are designed for grades three to eight. They test the four fundamental processes. They were designed and standardized by the Research Department of the Pittsburgh Public Schools. The three forms are of equal difficulty.

The following process steps are used in the Pittsburgh Scales.

<u>Addition</u>

1. Primary facts.
2. Decade drill.
3. Single column addition, with five addends.
4. Two place figures, with no carrying, gaps, or zeros.
5. Two place figures with carrying.
6. Columns, carrying, gaps, zeros.

<u>Subtraction</u>

1. Related facts.
2. One step borrowing.
3. More than one step borrowing.

<u>Multiplication</u>

1. Primary facts.
2. One place multiplier, no carrying.

---

1. <u>Pittsburgh Arithmetic Scales</u>, Public School Publishing company, Bloomington, Illinois, 1921

3. One place multiplier, carrying, requiring addition in higher decade.
4. One place multiplier, carrying, requiring additiion in higher decade.
5. Two or three place multipliers with carrying.
6. Single zero in multiplier.
7. Single zero in multiplier and in multiplicand.

Short Division

1. Primary facts.
2. Two or more digits in quotient, no carrying, no remainder.
3. Two or more digits in quotient, carrying, no remainder.

Long Division

1. Quotient difficulty. Trial divisor not correct quotient.

Tables XCV, XCVI , and XCVII . Addition facts and their frequency. Pittsburgh Arithmetic Scale Forms A,B,C.

Form A

	0	1	2	3	4	5	6	7	8	9
1							1			
2									1	2
3			1					1	2	1
4			1		2		1			1
5			1	1			1			
6				1						1
7							1		1	
8	1	1	1		1		1		1	
9			1	1			1		1	
10			1	1					1	1
11		1	1		1			1		1
12				1		1		1	1	
13								1	1	1
14				1						
15					1		1	2		1
16								1		
17						1	1		1	
18					1					1
19						1				2
20					1	1	1			
21							1		1	
22	2				1			1	1	
23							1		1	
24										3
25			1					1		
26										
27						1			1	
28				1			1	1		
29									1	2
30										
31							1	1		
32									1	
33			1		1			1	1	
34					1					
35			1		1			1		
37			1	1						
38					1	1				1
40			1				2			
41							1			
42						1				
43									1	
45										
46						1				
47							1			1
50										
52										
53		1								
57										

Form B

	0	1	2	3	4	5	6	7	8	9
1			1							1
2		2			1		1			1
3			2		1				1	
4		1	1			1	1			1
5		1	1	1			1	2		
6		1		1			1	1	1	
7				1			1			
8		1	1							
9			1							
10					1				1	2
11		1					1		1	
12					1		1	1		1
13						1	1			1
14							1			
15										
16										1
17				1						
18				1						1
19						1	1	2		
20			1							
21			1					1		
22								2		
23										
24										
25									1	1
26									1	1
27	1	1					1			
28								1		
29									2	
30										
31										
32										
33									1	
34										1
35			1				1			
37				1	1				1	
38										
40								1		
41						1			1	
42										
43									1	
45										
46			1							
47										
50										
52										
53										
57										

Form C

	0	1	2	3	4	5	6	7	8	9
1						1				
2		1	1			2			1	
3		2	1							
4			1	1	1		1		1	3
5							1		1	
6		1		1		1		1	1	1
7	1	1				1	1		2	1
8										
9				1	1					
10					1					1
11					1				1	
12				1				1		
13						2		2	3	1
14			2							
15			1			1				3
16						1	1			1
17		1						1		
18			1						1	1
19										
20					1	1		1	1	
21	1					2		1	1	
22							1			1
23				1						
24							2		1	
25										1
26			1					2	1	
27				1						
28				1		1				1
29							1			
30			1				1		1	
31							1	1		
32				1			1			
33								1	1	
34										1
35	2					1				1
37					1	1				
38								2		1
40										
41										
42									1	
43									1	
45			1							
46										
47						1	1			
50									1	
52		1								
53										
57								1		

Pittsburgh Arithmetic Scale, Form A, B, C. Explanation of foregoing table.

The numbers at the left are added to those across the top of the graph. These facts were found by adding up the columns.

28 of the primary facts in addition are used in Form A, 30 in Form B, and 29 in Form C. 66 of the upper decade facts to 39 + 9 are tested in Form A, 42 in Form B and 62 in Form C. In the higher decade 9 of the addition facts used in carrying in multiplication are used in Form A, 5 in Form B, and 9 in Form C.

Table XCVIII Subtraction facts and their frequency.
Pittsburgh Arithmetic Scale, Form A.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	1									1								
1			1					1										
2									1									
3												1						
4													1					
5							2	1					1	1				
6								1							1			
7									1				1	1		1	2	
8									2	1				1	1		2	4
9									1	1						2	1	3

Table XCIX Subtraction facts and their frequency.
Pittsburgh Arithmetic Scale, Form B.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0																			
1								1											
2				2				1		2		1							
3								1				2							
4							1												
5						1							2	1					
6							3					1		1		1			
7								3				2			2	1	2		
8														2	1	1	1	2	
9										1							3		2

Table C Subtraction facts and their frequency.
Pittsburgh Arithmetic Scale Form C.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0										2								
1		1						1										
2			1	1		1	1	1										
3					1					1								
4							1											
5						1	1	3	1						1			
6										1				1				
7								1	1				1	1			3	
8								2					1				2	2
9											1		2	1		1	3	2

The numbers at the left are subtracted from those across the top of the table. The borrowing method of subtraction was used.
29 of the subtraction facts are used in Form A, 28 in Form B, and 33 in Form C.

Table CI Multiplication facts and their frequency. Pittsburgh Arithmetic Scale Form A.

	0	1	2	3	4	5	6	7	8	9
0	1						1	3	4	2
1										
2							1	2	1	1
3			1	1			1	1		
4						1	1	1		
5								1	2	
6										
7					1	1	1	2	5	1
8	1						1	1	1	1
9	1						2	2	1	4

Table CII Multiplication facts and their frequency. Pittsburgh Arithmetic Scale, Form B.

	0	1	2	3	4	5	6	7	8	9
0	2		1			1		2	2	1
1										
2					1		1	1	1	1
3	1		1	1			1		1	
4					1		2	2	1	
5										
6	2			1			1	1	1	1
7	1		1		1		2	1	4	1
8	1				1	1	1	2	1	1
9			1		1		1	1	3	1

Table CIII Multiplication facts and their frequency. Pittsburgh Arithmetic Scale, Form C.

	0	1	2	3	4	5	6	7	8	9
3	1		2		1		1		1	1
5				1	1	1	1	3	1	
6				1		3	1		2	
7			2	1	3	2	4	2	5	2
8			1		1	1			1	1
9			1		1		1		1	1

The numbers at the left are multiplied by those across the top of the table.

34 multiplication facts are used in Form A, 46 in Form B, and 34 in Form C.

Tables CIV , CV , CVI Divisors and number of digits in dividends. Pittsburgh Arithmetic Scale Form A, Form B, Form C.

Form A

	37	56	63	85
4	1		1	
5		1		
9				1

Form B

	63	74	85	279
4	1			
5		1	1	
9				1

Form C

	68	73	279	299
4		1		
5	1			
6				1
9			1	

The numbers at the top of the tables are the actual divisors. The numbers at the left show how many digits are in the dividends.

Tables CIV, CV and CVI , show the actual divisors used and the number of digits in the dividends in Form A, B, and C.

Tables CVII , CVIII , and CIX Short Division facts and their frequency. Pittsburgh Arithmetic scale, Forms A, B, C.

Table Form A
Divisors

	3	5	7	9
3	1			
6	1	1		
9	1	1		
15		1		
42			1	
45		2		
46			1	
48			1	
49			1	
54				1
60			1	

Table Form B
Divisors

	6	8	9
30	1		
37			1
38		1	
45	1		
51			1
63			1
64		1	
66			1
81			1

Table Form C
Divisors

	2	3	4	7	8	9
0					1	
3		1				
4	1					
6	1	1				
8	1					
9		1				
24			1			
31			1			
32					1	
38			1			
42				1		
49				1		
60				1		
75					1	

The figures at the top are divided into those at the left of the table. The figure in the square denotes the frequency.
13 of the short division facts are used in Form A, and 9 in Form B, and 15 in Form C.

Table CX Number of facts Used in the Pittsburgh Arithmetic Scales.

Forms	A	B	C
Primary Facts Addition	28	30	29
Upper Decade Addition	66	42	62
Higher Decade Addition	9	5	9
Subtraction facts	29	28	33
Multiplication	34	46	34
Short Division	13	9	15

From Table CX it may be readily seen that the Pittsburgh Arithmetic Scales do not adequately tests the facts in the fundamental processes. They go beyond social usage[1] in addition, subtraction, multiplication, and short division. They are, therefore, of doubtful value.

1. Third Yearbook Department of Superintendence, Chapter 3, page 35

Fourth Yearbook Department of Superintendence, Washington, District of Columbia, 1926, page 177

Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin Company, New York, 1926, Chapter XIII, page 118

Table CXI Number of facts used in the fundamental processes of the Survey Tests.

Survey Tests	100 Primary Addition Facts	300 Upper Decade Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
Cleveland						
Form 1	90	143	53	79	85	91
Form II	89	162	55	70	81	94
Compass Elementary						
Form A	58	20	4	54	100	42
Form B	50	26	3	54	100	46
Advanced						
Form A	40	28	0	26	37	7
Form B	38	31	0	24	42	10
Compass Diagnostic						
Form A	79	81	2	58	77	49
Monroe Scale 1						
Form 1	80	54		68	84	90
Scale II						
Form I	63	96		54	71	
Scale 1						
Form II	76	54		60	81	92
Scale II						
Form II	56	98		51	66	
Scale I						
Form III	75	61		60	81	89
Scale II						
Form III	61	95		44	66	

Table CXI Number of facts used in the fundamental processes of the Survey Tests, continued.

Survey Tests	100 Primary Addition Facts	300 Upper Decade Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
Monroe Diagnostic						
Part 1	75	103	0	21	90	49
Part II	25	84	41	54	64	
Pittsburgh Scale						
Form A	26	66	9	29	34	13
Form B	30	42	5	28	46	9
Form C	29	62	9	33	34	15

From the data of Table CXI it may be seen that the Cleveland Survey Tests afford the most satisfactory survey of useful facts in the fundamental processes. In Form I, 90 of the primary addition facts, 143 of the upper decade facts, and 53 of the higher decade facts are used. 79 of the primary subtraction facts, 85 of the multiplication facts, and 91 of the short division facts are used.

In Form II, 89 of the primary addition facts, 162 upper decade facts, and 55 higher decade facts are used; 70 primary subtraction facts, 81 multiplication facts, and 94 short division facts are used. The other survey tests analyzed do not give as complete an inventory of useful facts.

The Cleveland Survey Tests are spiral in character, that is, the examples increase in difficulty, for example, Set A tests knowledge of the addition facts, Set E single column addition; Set J, more difficult column addition; Set M, carrying in column addition. In each of the fundamental processes the first set of examples is simpler, and each succeeding set grows more difficult, thus enabling the teacher to determine whether a lack of knowledge of the useful facts, primary and related, or a lack of comprehension of the process involved prevents the pupil from attaining satisfactory achievement.

The Cleveland Survey Tests conform more closely to social usage and to the grade requirements recommended by the Arithmetic Committee of the National

Education Association than the other survey tests analyzed. By omitting the work in fractions, Set O and single column addition with thirteen addends, Set J, the Cleveland Survey Tests conform to the demands of social usage, and are to be recommended for the purpose of a general survey.

Chapter V

Inventory Tests

a. Content and evaluation

The purpose of an inventory test in arithmetic is to enable the teacher to discover what combination each child is missing, or what type of example each child has difficulty in solving. This is especially necessary in the initial learning stages, where the pupil is apt to become confused by slight differences in the skills involved in the solution of particular types of examples. It is possible that a pupil be able to add simple addition combinations and short serial columns of addition, but be unable to add correctly a column which necessitates a long attention span and involves carrying.

Theoretically an inventory test covers completely the useful facts, primary and related, in the fundamental processes, and deals with each significant type of difficulty in each operation separately.

A teacher who is conscious of the total number of useful facts, primary and related, and of the types of examples in each process can

use an inventory test as a guide in instruction and prescribe the necessary remedial work for each individual pupil. When a child knows which combination or type of example is responsible for his incorrect work he endeavors to master his particular difficulty.

Authors differ in their conception of the significant process step difficulties. In analyzing the tests the process step difficulties used by each author have been listed.

a. The Buswell-John Diagnostic Chart for Individual Differences[1] in the fundamental processes in arithmetic was prepared after the work of two hundred and fifty elementary school pupils was observed. Afterwards these habits of work were arranged according to the frequency and an experimental form of diagnostic chart was printed. The experimental form was then used in seventy classrooms in ten different schools of Chicago and its suburbs. The habits listed in the extension of the experiment were combined with those listed in the original study of two hundred and fifty pupils and the present chart represents the frequency of each determined by the entire experiment. For example the most common habit observed in addition was error in making combinations. The list of habits in the diagnostic chart includes all habits which were observed in five or more children. The same set of examples are given in the Work Sheet as in the Diagnostic Chart.

---

1. Buswell, G.T., and John, Lenore, Diagnostic Chart for Fundamental Processes in Arithmetic, Public School Publishing Company, Bloomington, Illinois

Process Steps Used in Buswell-John Chart

Addition

1. Primary combinations.
2. Decade drill.
3. Short columns, one place figure, four addends.
4. Two or three place numbers with no carrying zeros, or gaps.
5. Same as Number 4 but with zeros or gaps.
6. Two or three place addends, carrying, sum of left hand column less than 10, no zeros.
7. Columns carrying, gaps, zeros.

Subtraction

1. Primary facts.
2. Related facts.
3. Simple subtraction without borrowing.
4. Borrowing, one step.
5. One-step borrowing. Last subtraction zero not brought down.
6. Double or triple borrowing.
7. Zero difficulties in minuend.

Multiplication Process Steps

1. Primary facts.
2. One place multiplier, no carrying.
3. One place multiplier, no carrying, requiring addition in higher decade.
4. One place multiplier, zero in multiplicand with and without carrying in zero.
5. Two or three place multipliers, no carrying.
6. Same as Number 5 but with carrying.
7. Single zero in multiplier, or multiplicand.
8. Double zero in multiplicand or multiplier.

Long Division Process Steps

1. Involving chiefly the form of operation, no carrying in multiplication, no borrowing in subtraction.
2. Carrying, borrowing.
3. Quotient difficulty.
   Trial divisor not the true quotient.
4. Quotient difficulty even though the increase by one rule is used.

Table CXIII Addition facts and their frequency. Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic.

	0	1	2	3	4	5	6	7	8	9
0								1		
1		5	4	3	1	6	2	6	9	2
2			3	1	1	5	4	0	3	4
3		1	1	2	9	1	1	1	1	0
4	2	1	0	1	3	2	0	0	2	1
5				1	2	1			1	
6		1	3				4	1		
7	1	1	1	1			2	5	3	4
8	1	2	1	6	2	2			4	2
9	2	1	3	2	1			1	4	1
10			2	1		3				
11		1							1	1
12	1			1	1	1		1	1	1
13					1	1				2
14				2					1	
15				2					1	3
16	2							1	1	
17		1					2	1	2	
18				1	1					
20									1	

	0	1	2	3	4	5	6	7	8	9
21							1			
22							3			
23									1	1
24	1			1			2			1
25							2			
27		1						1		
28									1	1
30							2			
32							1			
33									1	
36							1		1	
42		1								
44										1
53					1					
57										1
66								1		
73			1							
75							1			
76										
77										

The numbers on the left are added to those at the top of the table. The numbers in the squares show the number of times each group appeared, for example, 0 plus 7 appeared once, 1 plus 1 appeared 5 times. The facts are found by adding up the column of figures.

The facts in addition are not well covered. Only 69 primary facts are used, 49 upper decade facts, and 7 higher decade facts used in carrying in multiplication are used.

Table CXIV Subtraction facts and their frequency. Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0				1	1					2									
1		2		2	1	1	1	2		1									
2				2	1		1	2		4	4	1							
3				1	1	2	1		3	1		1	2						
4					1	1	1	1	3	1		1	2	1					
5							1		2	1			2						
6									1	1	1		4	2	1				
7								1		3		1			1		1		
8									3	1		2	1	2	2	1	1		
9										1	1		2			2		1	2

The numbers at the left are subtracted from those at the top of the table. The borrowing method of subtraction was used. The numbers in the squares indicate the number of times each group appeared, for example, 0 from 3 appeared once, 5 from 8 appeared twice.

The subtraction facts are not adequately tested. Only 63 facts are used.

Table CXV Multiplication facts and their frequency. Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic.

	0	1	2	3	4	5	6	7	8	9
0	5	1	7	3	3	7	6	2		4
1	1	1	2	1		3	1	2		1
2	1	4	2	6	7	3		2	2	2
3	1	4	6	4	1	5				
4		1	3	1	1					
5	1	2	2	1	5	2			1	
6	3	1	3	2	2	3	1	3		2
7	2	1		1	2	1	3	2	4	4
8	1			1	2		6	4	3	3
9	4		2		3	1	6	3	3	3

The numbers at the left are the multipliers, those at the top of the table are the numbers to be multiplied. The numbers in the square indicates the number of times the combination appeared, for example, 0 times 0 appeared 5 times, 6 times 3 appeared 2 times.

The facts in multiplication are more adequately tested than the facts in addition and subtraction. 76 facts are tested.

Table CXVI Facts used in Short Division and their frequency. Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic.

	1	2	3	4	5	6	7	8	9
0		3	3			1		1	
1		1				1			
2					2				
3			2			2			
4		3		1			1		
5					1				
6		3	2		1				
7							1		
8	1	2		1				1	
9		2	3						
10			3						
12			1						
14			1						
15			1		2				
16		1		1					
17			1						
18			2			1			
19								1	
20						1			
23			1						
24						1			
28			1						
31							1		
35							1		
36						1		1	
41									1
42							1		
45								1	
48						1			
50								1	
56								1	1
70									1
80									4

51 short division facts are tested.

Table CXVII Divisors and number of digits in dividends. Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic.

	16	17	21	25	34	37	46	78	395	400	530	924
3	1	1		1								
4			1									
5					1	1		1				
6							1		1	1	1	1

The numbers at the top of the table are the divisors. The numbers at the left show how many digits are in the dividends.

Table CXVIII Number of Facts Used in the Buswell-John Diagnostic Chart.

100 Primary Addition Facts	300 Upper Decade Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
69	49	7	63	76	52

The data of Table CXVIII shows that the Buswell-John Diagnostic Chart gives only a sampling of the facts used in the fundamental processes. The tests contain many process steps and are in line with the main purpose of testing in arithmetic, namely to give each child the greatest possible help in improving his work but they do not adequately test the facts in the fundamental processes.

b. "The primary purpose of the Wilson Process Inventory and Diagnostic Tests is to improve teaching. Testing as such, although it will be accomplished by these tests, is a secondary consideration. It is being more and more recognized, however, that good testing must reinforce good teaching. Tests which measure merely for comparative purposes are hardly justified for regular and intensive use. These tests help the teacher and the pupil by locating the specific causes of process difficulties."[1]

These tests were developed as a part of the program of the Committee on Arithmetic of the National Education Association for one hundred percent accuracy in the simple tools of arithmetic. They have been tried out in New England in connection with the Massachusetts state-wide and New England contests.

"The process steps in connection with each test constitute a carefully worked and teaching scheme. In addition there are ten such steps; in

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1. Wilson, G.M., The Wilson Inventory and Diagnostic Tests in Arithmetic, A Brief Manual of Instructions, The University Publishing Company, 1126 - 1128 Q Street, Lincoln, Nebraska, 1928

subtraction there are 20; in multiplication, 10; in short division, 10; and in long division, 10. In difficulty, the steps move from the easier and simpler to the more difficult and more complex. -- In connection with each process, there are additional opportunities for checking a pupil's errors. In the addition test there are fifteen additional error difficulties, which, taken with the original 10 groups, gives a total of 25 checks upon the child. These errors are not theoretical errors. The lists have been carefully made up from the actual analysis of errors made by children in the Massachusetts and New England contests during the past two years.

The additional error difficulties in subtraction are 27, which with the 20 teaching steps give a total of 47 checks; in multiplication, there are 17 additional checks, which taken with the 10 group teaching steps make a total of 27 checks; in short division, there are 27 additional error checks, which with the 10 group teaching steps, make a total of 37 checks; in long division, there

are 45 additional error checks, which with the 10 group teaching steps, make a total of 55 checks."[1] There are no time limits.

The process teaching step-difficulties are as follows:

Addition

1. Primary combinations, sums to 9 + 9.
2. Upper decade facts, sums to 39 + 9.
3. Short columns, unseen addends, no zeros, sums to 18.
4. Short columns, unseen addends, with zeros, sums to 18.
5. Two place or three place numbers with no carrying, no zeros, or gaps.
6. Two place or three place numbers without carrying but with zeros and gaps.
7. Two or three place addends, both two and three place, carrying, sum of left hand columns less than 10, no zeros.
8. Columns, carrying, gaps, zeros, sums to 39 + 9.

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1. Wilson, G.M., The Wilson Inventory and Diagnostic Tests in Arithmetic, A Brief Manual of of Instructions, The University Publishing Company 1126 -1128 Q Street, Lincoln, Nebraska, 1928

9. Adding dollars and cents.
10. Higher decades needed for carrying in multiplication.

Subtraction

1. Primary facts.
2. Related short division facts.
3. Simple subtractions, without borrowing.
4. Two or three place numbers, no borrowing, zero in answer.
5. Two or three place numbers, no borrowing, last subtraction a zero not brought down.
6. Two or three place numbers, no borrowing, zeros in subtrahend.
7. Gaps, or dangling lefts, no borrowing.
8. Two or three place numbers, one step borrowing.
9. Zero in the minuend and one step borrowing.
10. One step borrowing, the last subtraction a zero not brought down.
11. Gaps, or dangling lefts, with borrowing.
12. Double borrowing.
13. Zeros in the minuend and double borrowing.
14. Double borrowing, involving the figure nine in the subtrahend.

15. Vanishing lefts, and borrowing.

16. Vanishing lefts with double or triple borrowing.

17. Dollars and cents, zero difficulties, no borrowing.

18. Dollars and cents with zero difficulties and one step borrowing.

19. Dollars and cents with double or triple borrowing, zeros, gaps, and vanishing lefts.

20. Checking answers.

<u>Multiplication</u>

1. Primary combinations, no zeros.

2. Primary combinations, including zeros.

3. One place multiplier, no carrying (dollars and cents).

4. One place multiplier, carrying requiring addition in higher decade (dollars and cents).

5. One place multiplier, carrying requiring addition in same decade (dollars and cents).

6. One place multiplier, zeros in multiplicand with and without carrying into zero (dollars and cents).

7. Two and three place multiplier, no carrying (dollars and cents).

8. Two or three place multiplier, with carrying (dollars and cents).
9. Single zeros in multiplier (dollars and cents).
10. Double zeros in multiplicand or multiplier (dollars and cents).

Division

1. Primary combinations, to 81 ÷ 9, no remainders.
2. Uneven combinations, to 89 ÷ 9.
3. Two or more digits in quotient, no carrying, no remainders.
4. Two or more digits in quotient, no carrying but with remainders.
5. Two or more digits in quotients, with carrying, no remainders.
6. Two or more digits in quotient, with carrying, and with remainders.
7. One and two zeros in quotient, no carrying, no remainders.
8. One and two zeros in quotient, with carrying.
9. One or more zeros in quotient, with carrying and with remainders.
10. Final zero in quotient, with and without remainders.

Table CXIX Addition facts and their frequency. Wilson Inventory and Diagnostic Tests in Arithmetic.

	0	1	2	3	4	5	6	7	8	9
0	2	1	2	2	2	2	2	2	2	2
1	1	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2
3	2	2	2	2	2	2	2	2	2	2
4	2	2	2	2	2	2	3	2	2	2
5	2	2	2	2	2	2	2	2	2	2
6	2	2	2	2	3	2	2	2	2	2
7	3	2	2	2	1	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	1	2	2	2	1	2	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1
12	1	2	2	2	2	2	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1
14	1	2	2	2	2	2	2	1	1	1
15	1	2	2	2	2	1	1	1	1	1
16	1	2	2	2	2	2	2	1	2	1
17	1	1	1	1	1	1	1	1	1	1
18	1	2	2	2	2	2	2	2	2	1
19	1	1	1	1	1	1	1	1	1	1
20	1	2	2	2	2	1	1	1	1	1
21	1	2	2	2	2	2	2	1	1	1
22	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1
24	1	2	2	2	2	2	2	2	1	1
25	1	2	2	2	2	1	1	1	1	1

	0	1	2	3	4	5	6	7	8	9
26	1	1	1	1	1	1	1	1	1	1
27	1	2	2	2	2	2	2	2	2	1
28	1	2	2	2	2	2	2	1	1	1
29	1	1	1	1	1	1	1	1	1	1
30	2	1	2	1	2	2	1	1	1	1
31	1	2	1	1	1	1	1	1	1	1
32	1	2	3	2	2	2	2	2	1	1
33	1	1	1	1	1	1	1	1	1	1
34	1	1		1	1	1	1	1	1	1
35	1	2	2	2	2	2	2	1	1	1
36	1	2	2	2	2	2	2	2	2	1
37	2	1	1	1		1	1	1	1	1
38	1	1	1	1	1	1	1	1	1	1
39	1	1	1	1	1	1	1	1	1	1
40		1	1	1	2	1	1	1		
42		1	1	2	1	2	1			
45		1	1	1	2	1		2	2	
48		1	1	1	1	2	3	2		
49		1	1	1	2	2	2			
54		1	1	1	1	1	2	2	2	
56		1	1	2	1	1	1	2		
63		1	1	1	1	1	2	2	2	
64		1	1	1	1	1	1	2		
72		1	1	1	1	1	1	2	2	
81		1	1	1	1	2	1	2	1	
82										

The figures on the left are added to those across the top of the table. All the primary addition facts are used, 298 of the decade combinations and 79 of the upper decade facts used in carrying in multiplication.

Table CXX Addition facts and their frequency. Addition Process Step Difficulties. Wilson Inventory and Diagnostic Tests.

	0	1	2	3	4	5	6	7	8	9
0	3	1	2	1	1	2	1	1	2	1
1	3	4	5	5	6	2	3	2	4	3
2	1	5	5	3	4	6	4	5	4	4
3	3	2	6	2	3	4	5	3	3	3
4	2	7	4	3	2	4	1	5	1	2
5	2	3	6	5	3	2	2	2	2	1
6	1	3	4	4	2	2	3	2	3	2
7	3	8	4	1		2	2	2		1
8	2	4	2	2	1	3	1	3	3	2
9	4	1	2	1	2	2	4	4	1	2
10	3	1	1	2	1	4	2	2	3	2
11		1	2	2	2	2	2	3	3	3
12		2	1	1	2	1	2	1	1	1
13	1	2	1	1	2	2	1	2	1	3
14	1	1	1	2	2	2	1	2	1	1
15	1	1	2	2	1	3	2	2	1	1
16	2	2	2		1	2	1	1		1
17		1		1	1	1	1	1		
18		1		1	2	1	1	1	1	1
19			2	1	2		1	1	2	1
20	1	1	2	1		1	1	1	1	1
21				1	2	1	1		1	1
22	1				1	1	1	2	1	1
23			1					2	1	
24					1		1		1	
25	1					1	1	1		
26	1				1	1		1	1	
27				1				1	1	
28					1	1		1		
29							1	1	2	
30			1				2	1		
33		1								
34		1								
37										1
40						1				
42				1						
45				1	2					
48		1					1			
49					1	1	1			
54			1						1	
56						1		2		
63						1			1	
64							1			
72							1			
81								1		
82										

The figures on the left are added to those across the top.
98 of the primary facts in addition, 139 of the upper decade facts, and 18 of the higher decade facts used in carrying in multiplication are used in the tests devoted to process step difficulties in addition.

Table CXXI Subtraction facts and their frequency. Wilson Inventory and Diagnostic Tests.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	2	2	2	2	2	2	2	2	2	2									
1		2	1	1	1	2	2	2	2	2	2								
2			2	1	1	2	2	2	2	2	2	2							
3				2	1	2	1	2	2	2	2	2	2						
4					1	1	2	2	2	2	3	2	2	2					
5						2	1	2	2	2	2	3	3	2	2				
6							2	2	2	2	2	2	2	2	3	2			
7								2	2	2	2	2	2	2	3	2	3		
8									2	2	2	2	2	2	3	2	3	2	
9										2	2	2	2	3	2	2	3	2	3

The figures on the left are subtracted from those across the top. All the subtraction facts are used.

Table CXXII Subtraction facts used in Test 4P Process Step Difficulties. Wilson Inventory and Diagnostic Tests.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	6	1	3	2	1	3	2	1	2	2									
1		4	8	4	5	2	2	1	1	1	1								
2			4	5	4	5	3	3	4	1	3	1							
3				2	5	1	5	5	3	6		2							
4					4	5	1	4	3	2	2	1	1						
5						3	3	1	2	5	6		2	3					
6							3	2	1	6	1	1	1	1	1	3			
7								1	2	1		1	2	2	2	6	1		
8									2		3		1	2	2	1	5	4	
9										2	4	2	2		2	2	2	3	7

The figures on the left are subtracted from those across the top . 91 subtraction facts are used in Test 4P alone.

Table CXXII Multiplication facts and their frequency. Wilson Inventory and Diagnostic Tests.

	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	2	1	1	1		1	1	1	1	1

The numbers on the left are the multipliers. 99 of the multiplication facts are used.

Table CXXIII Multiplication facts and their frequency. Test 5P Process Step Difficulties. Wilson Inventory and Diagnostic Tests.

	0	1	2	3	4	5	6	7	8	9
0	15	5	6	4	8	11	6	7	9	9
1	5	1	5	4	4	2	4	2	4	2
2	7	7	9	10	7	8	3	3	3	4
3	4	3	5	4	4	2	5	5	5	1
4	6	4	5	2	5	6	4	4	4	5
5	4	5	2	1	3	4		3	3	4
6	6	2	3	2	4	4	4	4	5	3
7	2	2	1	2	2	4	4	3	5	1
8	3	2	2	4	1	2	1	2	3	2
9	4	1	1	3	3	3	3	3	2	3

The figures on the left are the multipliers. 99 of the multiplication facts are used in Test 5P alone.

Table CXXIV Short division facts and their frequency. Wilson Inventory and Diagnostic Tests.

	1	2	3	4	5	6	7	8	9
0		1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1
10		1	1	1	1	1	1	1	1
11		1	1	1	1	1	1	1	1
12		1	1	1	1	1	1	1	1
13		1	1	1	1	1	1	1	1
14		1	1	1	1	1	1	1	1
15		1	1	1	1	1	1	1	1
16		1	1	1	1	1	1	1	1
17		1	1	1	1	1	1	1	1
18		1	1	1	1	1	1	1	1
19		1	1	1	1	1	1	1	1
20			1	1	1	1	1	1	1
21		1	1	1	1	1	1	1	1
22			1	1	1	1	1	1	1
23			1	1	1	1	1	1	1
24			1	1	1	1	1	1	1
25			1	1	1	1	1	1	1
26			1	1	1	1	1	1	1
27			2	1	1	1	1	1	1
28			1	1	2	1	1	1	1
29			1	1	1	1	1	1	1
30				1	1	1	1	1	1
31				1	1	1	1	1	1
32				1	1	1	1	1	1
33				1	1	1	1	1	1
34				1	1	1	1	1	1
35				1	1	1	1	1	1
36				1	1	2	1	1	1
37				1	1	1	1	1	1
38				1	1	1	1	1	1
39				1	1	1	1	1	1
40					2	1	1	1	
41					1	1	1	1	2
42					1	2	2	1	1
43					1	1	1	1	1
44					1	1	1	1	1
45					1	1	1	1	2
46					1	1	1	1	1
47					1	1	1	1	1
48					1	2	1	2	1
49					1	1	2	1	1
50						1	1	1	1
51						1	1	1	1
52						1	1	1	1
53						1	1	1	1
54						2	1	1	2
55						1	1	1	1
56						1	2	2	1
57						1	1	1	1
58						2	1	1	1
59						2	1	1	1
60							1	1	1
61							2	1	1
62							2	1	1
63							2	1	2
64							2	2	1
65							2	1	1
66							2	1	1
67							2	1	1
68							2	1	1
69							2	2	1
70								2	1
71								2	1
72								2	2
73								2	1
74								2	1
75								2	1
76								2	1
77								2	2
78								2	2
79								2	2
80									2
81									1
82									2
83									2
84									2
85									2
86									2
87									2
88									2
89									2

The figures at the top are the divisors. 447 short divisor facts are used.

Table CXXV Short division facts and their frequency. Process Step difficulties. Wilson Inventory and Diagnostic Tests.

	2	3	4	5	6	7	8	9
0	2	2	5	4	7	3	5	7
1			2	2	1	1	1	2
2	2	1	1	1	1			
3	2	3			2	1	1	
4		1	2	1	2	1		1
5	1		2	3				1
6		8	1	1	4			1
7			1		3	4	1	
8	1		3	1		4	3	
9		2	1	1		2	1	2
10	2			2	1			
11	1	2		1	1			
12	1	1			2			
13						1		
14			1			6		
15		2		4	1	1		1
16	1	1	2			1	2	
17			1					1
18		1		1	1		2	5
19	1		1					
20			1		1			
21						3		
22							1	
23								1
24			2				1	
25				1				
26		2				1		

	2	3	4	5	6	7	8	9
27	3		1				1	
28	1		1			3		2
30				1	1			
32			1					
35				1				
36			1		2			2
37			2					
38					1			
40				1				
42				1	4	1		
45				2				1
47					1			
48				1	1		2	1
49				1		1		
52					1			
54					2			3
55								1
57							1	
63						4		
64							2	
65							1	1
72							1	1
81								2
85								1
89								2

The figures across the top of the table are the divisors. They are divided into the figures at the left. The figures in the squares denote the frequency. 140 of the short division facts are used in the tests, giving practice in process step difficulties.

Table CXXVI Divisors and the number of digits in the dividends. The Wilson Inventory and Diagnostic Tests.

	14	21	23	26	28	30	31	32	34	36	41	42	45	47	51	52
3	1	1			1	1		1			1					
4		1	1				1	2	1		1	1	2	2	1	2
5				1				1								
6							1									

	54	59	61	63	64	66	67	70	71	72	73	74	81	83	91	98
3										1						
4	1	1	2	1	1	1	1	1	1		2	1	1		1	1
5					1									1		
6												1				

	111	121	131	222	1122
3					
4	1		1		
5				1	
6		1			1

The figures at the top of the table are the divisors, those on the left are the number of digits contained in the dividends.

Table CXXVII Number of facts used in the fundamental processes. The Wilson Inventory and Diagnostic Tests.

100 Primary Facts in Addition	300 Upper Decade Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
100	298	79	100	99	447
Facts Used in Process Step Difficulties					
98	139	18	91	99	140

From the data of Table CXXVII it may be seen that the Wilson Inventory and Diagnostic Tests in Arithmetic give an almost perfect inventory of facts in the fundamental processes. They provide a detailed analysis of arithmetic skills. For example, the tests in addition contain ten types of examples each of which differs from the others because of a difference in the skills involved in addition. The tests in each process are constructed on the same basis of analysis of difficulties.

The Wilson Inventory and Diagnostic Tests in Arithmetic are essentially inventory tests as may be seen from the careful inventory of facts and

processes. They are valuable to the teacher because they point out which of the combinations each pupil knows and does not know; they discover what skills have or have not been mastered, and indicate where future instructional emphasis should be placed. The tests conform to social usage. They are in harmony with and reinforce the right curricular principles. They reinforce proper methods of teaching, therefore they are to be highly recommended for use in the classroom.

c. Accuracy is obtained only by drill in the combinations which arise in each of the fundamental processes. Instead of presenting a few of the fundamental combinations the Wisconsin Inventory Tests[1] endeavor to present all of them. These tests were used in the Third Annual Nation-Wide Survey. "The Wisconsin Inventory Tests show that only a few pupils in the poorest classes miss more than 10 combinations out of 100 on the inventory. This means that even in the poorest and most discouraging class that any teacher has, all but a few of the pupils are at least 90% perfect. Bearing this in mind and remembering that these children will learn the remaining 10% of the combinations when they know what they are, we have every reason to feel assured that the long standing difficulties in the fundamentals of arithmetic can be conquered with ease and without loss of time. The inventory technique is the only known device

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1. Wisconsin Inventory Tests in Arithmetic, Public School Publishing Company, Bloomington, Illinois

that will produce results of this sort, It also points to a fundamental but much neglected principle of teaching. Show the pupil just what he knows and just what he does not know, and he will do the rest."[1]

The plan of the tests is as follows:

Test 1 The 100 First Decade Combinations in Addition.

Test 2 The 100 Fundamental Combinations in Subtraction.

Test 3 The 100 Combinations in Multiplication.

Test 4 Short Division Combinations.

Table 5 The Most Useful Combinations in Higher Decade Addition.

Test 6 The Addition Combinations Needed for Carrying in Multiplication.

Test 7 Combinations which give Zero Quotients in Short Division.

Test 8 Long Division.

Test 9 Bridging in the Addition of Mixed Numbers.

Test 10 Bridging in the Subtraction of Mixed Numbers.

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1. Osborn, W.J., Teacher's Handbook for the Wisconsin Inventory Tests in Arithmetic, page 4

Test 11 One Step Problems.

Test 12 Denominate Numbers.

Table CXXVIII Addition facts and their frequency. Wisconsin Inventory Tests.

	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1
10		1	2		2	1	1	1	1	1
11		1	1	1		1	1	1	1	1
12		1	1	1	1	1	1	1	1	1
13		1	1	1	1	1	1	1	1	1
14		1	1	1	1	1	1	1	1	1
15		1	1	1	1	1	1	1	1	1
16		1	1	1	1	1	1	1	1	1
17		1	1	1	1	1	1	1	1	1
18		1	1	1	1	2		1	1	1
19		1	1	1	1	1		1	1	1
20		1	1	1	1	1	1	1	1	1
21		1	1	1	1	1	1	1	1	1
22		1	1	1	1	1	1	1	1	1
23		1	1	1	1	1	1	1	1	1
24		1	1	1	1	1	1	1	1	1
25		1	1	1	1	1	1	1	1	1

	0	1	2	3	4	5	6	7	8	9
26		1	1	1		1	1	1	1	
27		1	1	1	1	1	1	1	1	1
28		1	1	1	1	1	1	1	1	1
29		1	1	1	1	1	1	1	1	1
30		1	1	1	1	1	1	1	1	1
31		1	1	1	1	1	1	1	1	
32		1	1	1	1	1	1	1		
33		1	1	1	1	1	1			
34		1	1	1	1	1				
35		1	1	1	1	1	1			
36		1	1	1	2	1	1	1	1	1
37		1	1							
38		1								
40		1	1	1	1	1	1			
45		1	1	1	1	1	1	1	1	
48		1	1	1	1	1	1	1		
49		1	1	1	1	1	1			
54		1	1	1	1	1	1	1	1	
56		1	1	1	1	1	1	1		
63		1	1	1	1	1	1	1	1	
64		1	1	1	1	1	1	1		
72		1	1	1	1	1	2		1	
81		1	1	1	1	1	1	1	1	
82										
83										
84										

The numbers on the left are added to those across the top of the table. The number in the square denotes the frequency.

100 primary facts are used in addition, 227 of the upper decade facts, and 79 of the higher decade facts used in carrying in multiplication. The addition facts are well covered.

Table CXXIX Subtraction facts and their frequency.
Wisconsin Inventory Tests.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	1	1	1	1	1	1	1	1	1									
1		1	1	1	1	1	1	1	1	1	1								
2			1	1	1	1	1	1	1	1	1	1							
3				1	1	1	1	1	1	1	1		1	1					
4					1	1	1	1	1	1	1	1	1	1					
5						1	1	1	1	1	1	1	1	1	1				
6							1	1	1	1	1	1	1	1	1	1			
7								1	1	1	1	1	1	1	1	1	1		
8										1	1	1	1	1	1	1	1	1	
9											1	1	1	1	1	1	1	1	1

The numbers at the left are subtracted from those across the top of the table.
98 of the facts in multiplication are used.

Table CXXV Multiplication facts and their frequency.
Wisconsin Inventory Tests.

	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1

The numbers at the left are multiplied by those across the top of the table.
The 100 facts in multiplication are tested.

Table CXXXI Short division facts and their frequency. Wisconsin Inventory Tests.

	1	2	3	4	5	6	7	8	9
0	1	1	1	1	2	1	2	3	2
1		1	1	1	1	1	1	2	2
2			1	1	1	1	1	1	1
3			1	1	1	1	1	1	1
4					1	1	1	1	1
5						2	1	1	
6								1	2
7	1							1	1
8	1							1	1
9	1	1							
10							1		
14		1							1
15									
16		1						1	
17									
18		1				1			1
20			1		1	1	1	1	1
21						1		1	1
22						1		1	1
23						1		1	1
24			1	1		1		1	1
25									1
26									1
27			1				1		2
28				1					
30				1		1	1	1	1
31				1			1	1	2

	1	2	3	4	5	6	7	8	9
32				1			1	1	1
33							1		1
34							1		1
35					1		1		1
36				1		2			1
38					1				1
39									
40					1	1		2	2
41						1	1		1
42						1	1		1
43									1
44									2
45					1				1
46							1		
48						1		1	
49							1		
50						1	1	1	1
51						1	1	1	1
52						1	1	1	1
53						1	1	1	1
54						1	1	1	2
55							1	1	
56							2	2	
60							1	1	1
61							1	1	1
62							1	1	1
63							2	1	1

The figures across the top of the table are the divisors. The figures in the squares denote the frequency, for example, 63 is divided by 7, twice, by 8, once, by 9, once.

162 short division facts are used.

Table CXXXII Divisors and number of digits in dividends. Wisconsin Inventory Tests.

	48	51	61	68	70	72	87	312	352
4	1	1	1	1	1	1			
5							1		
6								1	
7									1

The numbers at the top of the table are the divisors, those on the left are the number of digits contained in the dividends.

Table CXXXIII Fractions to be changed to whole or mixed numbers. Wisconsin Inventroty Tests.

2/2	4/4	7/5	8/6	7/7	10/8	21/8	16/9	20/12	22/16
3/2	5/4	8/5	9/6	8/7	11/8	22/8	17/9	21/12	23/16
4/2	6/4	9/5	10/6	9/7	12/8	23/8	12/12	22/12	24/16
5/2	7/4	10/5	11/6	10/7	13/8	9/9	13/12	23/12	25/16
3/3	8/4	11/5	12/6	11/7	14/8	10/9	14/12	16/16	26/16
4/3	9/4	12/5	13/6	12/7	15/8	11/9	15/12	17/16	27/16
5/3	10/4	13/5	14/6	13/7	17/8	12/9	16/12	18/16	28/16
6/3	11/4	14/5	15/6	18/7	18/8	13/9	17/12	19/16	29/16
7/3	5/5	6/6	16/6	8/8	19/8	14/9	18/12	20/16	30/16
8/3	6/5	7/6	17/6	9/8	20/8	15/9	19/12	21/16	31/16

Table CXXXIV Mixed numbers to be changed to improper fractions. Wisconsin Inventory Tests.

1 1/3	1 4/7	1 6/9	1 2/16
1 1/4	1 5/7	1 7/9	1 3/16
1 2/4	1 1/8	1 1/12	1 4/16
1 1/5	1 2/8	1 2/12	1 5/16
1 2/5	1 3/8	1 3/12	1 6/16
1 3/5	1 4/8	1 4/12	1 7/16
1 1/6	1 5/8	1 5/12	1 8/16
1 2/6	1 6/8	1 1/12	1 9/16
1 3/6	1 1/9	1 7/12	1 10/16
1 4/6	1 2/9	1 8/12	1 11/16
1 1/7	1 3/9	1 9/12	1 12/16
1 2/7	1 4/9	1 10/12	1 13/16
1 3/7	1 5/9	1 1/16	1 14/16

Table CXXXV Fractions and mixed numbers used in subtraction. Wisconsin Inventory Tests.

4 - 1 1/2	8 - 2 5/6	7 - 5 5/8	7 - 2 11/12
3 - 1 1/3	2 - 1/7	6 - 7/8	6 - 3 1/16
5 - 2/3	4 - 3 2/7	8 - 3 1/9	2 - 1 3/16
7 - 5 1/4	9 - 5 3/7	9 - 7 2/9	4 - 3 5/16
1 - 3/4	8 - 3 4/7	8 - 6 4/9	6 - 2 7/16
3 - 2 1/5	3 - 2 5/7	1 - 5/9	5 - 3 9/16
1 - 2/5	7 - 2 6/7	9 - 8/9	6 - 5 11/16
9 - 5 3/5	2 - 1 7/9	3 - 1 1/12	9 - 3 13/16
8 - 7 4/5	1 - 1/8	4 1 1 5/12	2 - 15/16
6 - 3 1/6	1 - 3/8	5 - 7/12	

Table CXXXVIDenominate numbers used in reduction, ascending and descending. Wisconsin Inventory Tests.

Column 1	Column 2
inches	feet
feet	inches
yards	feet
rods	yards
yards	rods
rods	miles
sq.in.	sq.ft.
sq.ft.	sq.in.
sq.ft.	sq.yds.
sq.yds.	sq.ft.
sq.rds.	sq.mi.
sq.mi.	acres
acres	sq.mi.
acres	sq.rds.
sq.rds.	acres
cu.in.	cu.ft.
cu.ft.	cu.in.
cu.ft.	cu.yds.
pints	quarts
quarts	pints
quarts	pecks
pecks	quarts
pecks	bushel
bushel	pecks
quarts	gallon
gallon	quarts
ounces	pounds
pounds	ounces
cents	nickels
nickels	cents
dimes	cents
cents	dimes
minutes	hours
days	weeks
weeks	days
months	years
years	months

The units of measure in column 1 are to be changed to their equivalents in column 2.

Wisconsin Inventory

<u>Subject Matter of Problems</u>

Buying toys, fruit.

Having certain number of articles and getting more.

Differences in age, hours, pounds.

Finding sum, product, quotient.

Number of quarts of berries picked.

Number of glasses of lemonade served.

Spending money, amount left.

Number of plants in a row.

Dividing candy among number of boys.

Amount of ice cream necessary to serve so many people.

Time to walk certain number of miles.

Length of time necessary to save a certain amount of money.

Weight of milk.

Table CXXXVII Classification of problems involving fundamental processes. Wisconsin Inventory Tests.

Classification	Frequency
Addition	13
Subtraction	16
Multiplication	15
Division	15
Fractions	1
Total	60

Table CXXXVIII Number of Facts Used in the Wisconsin Inventory Tests in Arithmetic.

100 Primary Addition Facts	300 Upper Decade Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
100	227	79	98	100	162

From the data on Table CXXXVIII it may be seen that the Wisconsin Inventory Tests in Arithmetic cover nearly all the facts in the fundamental processes. The tests are arranged according to carefully planned process step difficulties. They are, therefore, of great value in identifying pupil abilities and weaknesses. The work in fractions and denominate numbers is simple. The problems are a step in advance of the typical textbook problems. They deal with situations that are real and that children understand.

The Wisconsin Inventory Tests in the fundamental processes are in harmony with and reinforce the right curricular principles. They reinforce proper methods of study. Therefore they are to be recommended for classroom use.

Table CXXXIX Facts used in the fundamental processes in the Buswell-John Diagnostic Chart, Compass Diagnostic Tests, Monroe Diagnostic Tests, The Wilson Inventory and Diagnostic Tests, and the Wisconsin Inventory Tests in Arithmetic.

	100 Primary Addition Facts	300 Upper Decade Facts	80 Higher Decade Facts	100 Primary Subtraction Facts	100 Multiplication Facts	Short Division Facts
Buswell-John	69	49	7	63	77	52
Compass	79	81	2	58	77	49
Monroe	Part 1 75 Part 2 25	Part 1 103 Part 2 84	Part 1 0 Part 2 41	Part 1 21 Part 2 54	Part 1 90 Part 2 64	Part 1 49 Part 2
Wilson	100	298	79	100	99	447
Wisconsin	100	227	79	98	100	126

The Wilson Inventory and Diagnostic Tests, and the Wisconsin Inventory Tests in Arithmetic, afford the most satisfactory inventory of facts in the fundamental processes. The Wilson Inventory and Diagnostic Tests are slightly superior in testing more completely the number of fundamental facts in the fundamental processes, and much superior in the analysis of process step difficulties.

Diagnostic testing is aimed not only at finding deficiencies, and organizing remedial instruction, but it is quite as valuable for furnishing better insight to the teacher of arithmetic by acquainting her with what is involved in the learning process of arithmetic, and what types of examples offer greatest difficulty.

Chapter VI

Problem or Reasoning Tests

a. Content and evaluation

Problem work should consist largely in the selection and performances of computations that would be needed in a particular situation in which the pupil would need to locate and select the essential data himself instead of having it presented to him in an organized manner as is now done in verbal problems.

The pupils should learn in a natural situation such as they meet in life and should have an active part in the planning and execution of such activities to the end that they may develop understanding of business and develop judgment.

At present requirements for a satisfactory reasoning test are only approximated. The reasoning process is not mechanical. It tends to be individualistic and adaptable. This adds to the difficulty of measurement. All of the reasoning tests are general tests. They are not diagnostic except for the particular problem included.

a. The Stevenson Problem Analysis Test[1] consists of four tests each containing six problems. The tests endeavor to test ability to read problems, understand them, determine the process to be used, and approximate the answer. The following questions are asked for each problem:

A. Which of the following facts are given in the problem?

B. Which of the following things are you asked to find out in the problem?

C. Which of the following is the most reasonable answer?

D. Which process should be used in solving the problem?

It is an alternate response test, four statements being given under each question.

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1. Stevenson Reading Tests Problem Analysis, Public School Publishing Company, Bloomington, Illinois

Test 1 of the Stevenson Reading Test is designed for Grades IV - VI; Test II is designed for Grades VII - X.

Situation Involved in Problems

Form 1 Test 1

1. Cost of certain number of automobiles at $ — each.
2. Number of pounds in a given number of bags of potatoes.
3. Number of boxes needed to hold so many fancy apples.
4. Buying and selling a boat. Gain?
5. Boys gathered nuts. Number of nuts gathered.
6. Length of Mississippi River and number of miles navigable. Number of miles not navigable.

Form 1 Test 2

1. Reading gas meter.
2. Earning money working for father.
3. Wages of men working on building at same rate per day.
4. Buying vacuum cleaner on instalment plan.

5. Business man's personal expenditures.
   Salary given and itemized list of expenses.
6. Development Company buying land and dividing it into lots.

Form 2 Test 1

1. Finding given weight of children.
2. Buying football tickets.
3. Buying and selling a house.
4. Dividing money equally among five boys.
5. Buying a supply of coal.
6. Spending money per month.

Form 2 Test 2

1. Pupils making school badges.
2. Buying and selling farm.
3. Making toy boats and selling them.
4. Finding distance from one city to another.
5. Cost of dress figuring materials and certain value for time spent.
6. Cost of meal at restaurant.

The problems are typical of those found in textbooks of arithmetic and traditional classrooms. "The isolated written problem of the usual type

should disappear from textbooks, and the schoolroom. It should be replaced by the real problem arising directly out of the child's experience. This means that written problem work hereafter should be based upon large, sizable, thinking situations, and usually developed in the local community. They should be well within the comprehension of the child, and should appeal strongly to his interests. They are not primarily computational, but informational. They should develop understanding of business, and judgment in the expenditure or use of money."[1]

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1. Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin Company, Boston, Massachusetts, 1926, page 117 - 118

Chapter VII

Summary*

The need of adequate ways and means of testing instruction in arithmetic has been felt for many years. The pioneers in the field of testing in the fundamental processes were Rice and Stone. Courtis[1] came next.

Many tests in arithmetic have been prepared by those who had little familiarity with the teaching of arithmetic and who had given no particular study to the evaluation of its subject matter before undertaking their work. "Certain of the authors of these tests did possess, however, a thorough familiarity with statistical technique as applied to educative problems which they hastened to adapt to arithmetic without carefully selecting in advance the problems to which they applied it. The result is that these standardized tests contain considerable material which is not representative of the best practice in the teaching of arithmetic; which tends to standardize

1. Monroe, W.S., DeVoss, J.C., and Kelly, F.J., Educational Tests and Measurements, Houghton Mifflin Company, Boston, Massachusetts, 1924, page 18 - 19

* There is some repetition. It is for the convenience of the reader who may wish to turn at once to the summary.

topics which should not be taught, and which consequently handicaps certain efforts to reorganize the curriculum." [1]

The problem in this study is to analyze and evaluate standardized tests in arithmetic, and to so arrange and interpret the data that it may be found helpful in choosing or rejecting a standardized test.

Arithmetic is a tool subject, Perhaps no other subject in the entire school curriculum was affected so much by the abandonment of the theory of formal discipline as was the teaching of arithmetic. It is now a widely accepted aim that arithmetic should serve a socially useful purpose. By limiting the work in the elementary grades to those processes which are useful and by insisting upon 100% mastery of them, the needs of society shall be met. It is now generally accepted that formal procedure upon useless and often meaningless processes is

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1. Upton, Clifford B., _Influence of Standardized Tests on the Curriculum in Arithmetic_, Mathematics Teacher, April 1925, page 193 - 208

detrimental to a child's interest, and to his habits of study.

Research has shown that the greater part of arithmetic in common use consists of comparatively small numbers, and the problems involved are relatively easy of interpretation.[1]

In choosing a test in arithmetic, care should be taken that the test be in harmony with and reinforce the right curricular principles; and that it reinforces the proper method of teaching.[2]

Tests in arithmetic are given mainly for the purpose of securing information which can be used in making instruction more effective. Arithmetic tests may be divided into two classes: 1. Tests in abstract numbers. 2. Tests in written problems. The tests in abstract numbers divide themselves into two groups: 1. The survy type which consists of a sampling of the useful facts and processes, such as the Cleveland

1. Third Yearbook, Department of Superintendence, National Education Association, Washington, District of Columbia, Chapter III Arithmetic, Summary of Research Studies, page 35 - 109

2. Wilson, G.M., and Hoke, K.J., How to Measure The Macmillan Company, New York, 1929, Revised Edition, page 516

Survey or the Monroe Survey Tests. 2. The inventory test which theoretically is a complete inventory of the useful facts and processes such as the Wilson Inventory and Diagnostic Tests in Arithmetic. Much less attention has been devoted to the construction of tests in written problems than to tests in abstract numbers. Problem work should involve natural situations which the pupils encounter in their activities. The pupil should have an active part in the planning and execution of such activities that they may develop understanding of business and judgment in the use of money. No outstanding tests in written problems have been constructed.

The purpose of a survey test, as the name implies, is to secure a general measure of the level of achievement of a class or school. A survey test in arithmetic merely indicates that a class or school is below the standard. It does not indicate the specific reason for the deficiency.

The Cleveland Survey Tests originated in the educational survey of the Cleveland Public Schools. The material was based on the Cleveland course of study.

The tests are composed of fifteen different sets of examples, and are to be used in grades III to VIII. They aim to test the fundamentals of arithmetic. There are four sets of the tests in addition; two in subtraction; three in multiplication; four in division; and two in fractions. In each of the fundamental processes, the first set of examples is simple and each succeeding test increases in difficulty thus enabling the teacher to determine whether a lack of knowledge of the useful facts, primary and related, or a lack of comprehension of the process involved prevents the child from attaining satisfactory achievement. The main purpose of testing in arithmetic is to help the pupil. "We have come to realize that each separate response in the useful tool material of arithmetic must be mastered, and in turn tested if the diagnosis of the pupil's ability is to be complete. The present

tendency, therefore, in testing in arithmetic is to cover completely the operations in all of their specific phases and do this in such a manner that diagnosis of pupil weaknesses becomes relatively easy. That is, since we have now narrowed our work in arithmetic to the useful phases, it is unsatisfactory to ascertain merely the percentage of mastery by the use of a random sampling; what is wanted is a complete inventory of accomplishments and deficiencies."[1]

Of the survey tests, analyzed and evaluated, the Cleveland Survey Tests afford the most satisfactory inventory of useful facts, primary and related, in the fundamental processes. In Form I, 90 of the 100 primary addition facts, 143 of the 300 upper decade facts, and 53 of the 80 higher decade facts used in carrying in multiplication are tested. 79 of the 100 primary subtraction facts, 85 of the 100 multiplication facts, and 91 of the 449 short

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1. Wilson, G.M., and Hoke, K.J., How to Measure The Macmillan Company, New York, 1929, page 71

division facts are used. In Form II, 89 of the 100 primary addition facts, 162 of the 300 upper decade facts, and 55 of the 80 higher decade facts are used; 70 of the 100 primary subtraction facts, 81 of the 100 multiplication facts, and 94 of the 449 short division facts are used.

The Cleveland Survey Tests conform more closely to social usage, and to the grade requirements recommended by the Arithmetic Committee of the National Education Association than the other survey tests analyzed. By omitting, Set C, which consists of work in fractions, and Set J, which consists of single column addition with thirteen addends, the Cleveland Survey Tests in Arithmetic conform to the demands of social usage, and are to be recommended for the purpose of a general survey.

The Compass Survey Tests in Arithmetic are prepared in two forms, A and B. The Elementary Examination prepared for Grades II, III, and IV, consists of tests in addition, subtraction, multiplication, and division of whole numbers. The

Advanced Examination designed for Grades IV to VIII, consists of examples in whole numbers, fractions, decimals, denominate numbers, percentage, and general problems.

The Compass Survey Tests in Arithmetic are designed to test material in Grades II, III, and IV, that the National Committee on Arithmetic[1] recommends for higher grades. The following unreasonable grade requirements were noted:

Formal drill in Grade II

Long Division in Grade III.

Percentage, interest, and commission in Grades IV and V.

The useful facts, primary and related, in the fundamental processes are not adequately tested. In the Elementary Examination, 58 of the 100 primary addition facts, [illegible] of the 300 upper decade facts, and 4 of the higher decade facts used in carrying in multiplication are tested; [illegible] of the 100 primary subtraction facts, 100 of the 100 multiplication

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1. Third Yearbook, Department of Superintendence, National Education Association, Chapter III, Arithmetic, pages 35 - [illegible]

facts, and 42 of the 449 short division facts are tested. In Form B of the Elementary Examination 52 of the primary addition facts, 26 of the 300 upper decade facts, 3 of the 80 higher decade facts are tested; 54 of the 100 subtraction facts, and 46 of the 449 short division facts are tested.

In the Advanced Examination, Form A, only 40 of the 100 primary addition facts, 28 of the 300 upper decade facts, 26 of the 100 subtraction facts, 37 of the 100 multiplication facts, and 7 of the 449 short division facts are tested. In Form B of the Advanced Examination, 38 of the 100 primary addition facts, 31 of the 300 upper decade facts, 24 of the primary subtraction facts, 42 of the 100 multiplication facts, and 10 of the 449 short division facts are tested.

The examples in fractions go beyond social usage, such as addition of ninths and fifths; subtraction of ninths from thirds. There is little or no use for the multiplication of a fraction by a fraction. The case for division of fractions is even less favorable.[1]

1. Upton, Clifford B., Changing the Curriculum in Arithmetic, Teachers' College Record :341 - 359. December 1926

The examples in decimals and in denominate numbers consist of drill on useless and traditional material.[1] Most of the problems are the traditional problems of the classroom, involving situations about which the pupils know nothing.

The Compass Survey Tests in Arithmetic are not in harmony with nor do they reinforce the right curricular principles; they do not adequately test the useful facts, primary and related in the fundamental processes. The Compass Survey Tests in Arithmetic have little to recommend them.

The Monroe Standardized General Survey Arithmetic Scales are designed to test the four fundamental processes in whole numbers, in fractions, and in decimals. Scale 1 is for use in Grades III, IV, and V. It consists of eight tests. Scale II is for use in Grades VI, VII, and VIII. It consists of seven tests. There are three forms for each scale. All three forms are similar in content.

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1. Sala, V., Denominate Numbers Used in the Factories of New Britain, Connecticut, Boston University School of Education, Master's Thesis, 1931

The tests are apiral in arrangement thus presenting process steps to help the teacher determine on which specific skill in the process, the pupil fails.

In Scale I, Form I, 80 of the 100 primary addition facts, and 54 of the 300 upper decade facts are tested; 68 of the 100 primary subtraction facts, 84 of the 100 multiplication facts, and 90 of the 449 short division facts are tested. In Scale II, Form I, 63 of the 100 primary addition facts, and 96 of the 300 upper decade facts are tested; 54 of the 100 primary subtraction facts, and 71 of the 100 multiplication facts are tested. In Scale 1, Form II, 76 of the 100 primary addition facts, 54 of the 300 upper decade facts, 60 of the 100 primary subtraction facts, 81 of the 100 multiplication facts, and 92 of the 449 short division facts are tested. In Scale II, Form II, 56 of the 100 primary addition facts, and 98 of the 300 upper decade facts are tested; 51 of the 100 primary subtraction facts, and 66 of the 100

multiplication facts are tested. In Scale I, Form III, 75 of the 100 primary addition facts, 61 of the 300 upper decade facts, 60 of the 100 primary subtraction facts, 81 of the 100 multiplication facts, and 89 of the 449 short division facts are used. In Scale II, Form III, 61 of the 100 primary addition facts, 95 of the 300 upper decade facts, 44 of the 100 primary subtraction facts, and 66 of the 100 multiplication facts are tested.

The Monroe Standardized General Survey Scales afford a sampling of the useful facts in the fundamental processes. They go beyond social usage in addition, subtraction, multiplication, and division of fractions, and in division of decimals. Because of the above if the tests in fractions and in decimals are omitted the Monroe Standardized General Survey Scales may be used for the purpose of a general survey.

The Pittsburgh Arithmetic Scales are designed for grades three to eight. They are spiral in arrangement and aim to test the four fundamental processes.

In Form A of the Pittsburgh Arithmetic Scales, 26 of the 100 primary addition facts, 66 of the 300 upper decade facts, and 9 of the higher decade facts used in carrying in multiplication are used; 29 of the 100 primary subtraction facts, 34 of the 100 multiplication facts, and 13 of the 449 short division facts are tested. In Form B, 30 of the 100 primary addition facts, 42 of the 300 upper decade facts, and 5 of the 80 higher decade facts are tested; 28 of the 100 primary subtraction facts, 46 of the 100 multiplication facts, and 9 of the 449 short division facts are tested. In Form C, 29 of the 100 primary addition facts, and 9 of the 80 higher decade facts are used; 33 of the 100 primary subtraction facts, 34 of the 100 multiplication facts, and 15 of the 449 short division facts are tested. The Pittsburgh Arithmetic Scales do not adequately test the useful facts,primary and related in the fundamental processes. They go beyond social usage, for example, addition of five place figufes with 7 addends; subtraction of nine place figoues from 10 place figures; multiplication of

5 place figures by 3 place figures; division of 9 place figures by two place figures. The Pittsburgh Arithmetic Scales are of doubtful value to the teacher.

An inventory test is designed to point out the particular weaknesses of individual children. It Is in giving such detailed information that the inventory test is of value. Class or pupil weaknesses may be investigated with exactness. Once a difficulty has been diagnosed it can be remedied by proper instruction and practice. If a pupil cannot borrow but has mastered the primary facts in subtraction he can be given instruction and practice in borrowing, and may be excused from further drill in the primary subtraction combinations. Theoretically an inventory test is a complete inventory of the useful facts and processes.

The Compass Diagnostic Tests in Arithmetic do not meet the standards of inventory tests and therefore do not come under that classification. The Compass Diagnostic Tests consist of twenty different tests each dealing with some one of the processes or with problem solving. The method of diagnosis is to give a test in a single process to determine the

level at which the skill of the pupil in anelement of the process breaks down and thus aid the teacher inlocsting the cause of weakness in the process.

The Compass Diagnostic Tests do not adequately test the useful facts, primary and related in the fundamental processes. Only 79 of the 100 primary addition facts, 81 of the 300 upper decade facts, and 2 of the 80 higher decade facts are tested; 58 of the 100 primary subtraction facts, 77 of the 100 multiplication facts, and 49 of the 449 short division facts are tested. The Compass Diagnostic Tests afford only a sampling of the useful facts, primary and related in the fundamental processes and therefore may be classed as a survey test.

The Compass Diagnostic Tests go beyond social usage in fractions, such as: addition of fifths, sevenths, and thirty fifths; subtraction of twelfths from ninths; multiplication of twenty-fifths by sixteenths; and division of sixteenths by fortieths.[1]

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1. Fourth Yearbook, Department of Superintendence National Education Association, Chapter V Arithmetic page 177

In social usage, decimals are used mainly with United States money. The following examples do not refer to United States money.

1. 34.5 + .09 + 627 + 3.014

2. 36.724 - 36.0724

3. .9182 x .2104

4. 1.008 ÷ .04

The work in addition, subtraction, multiplication, and division of denominate numbers is obsolete.[1]

Examples in mensuration should be informational.[2] The aim of informational subject matter is defeated when subjected to formal drill and testing. The problems are the typical classroom problems. They present farfetched situations or deal with material with which the child is not familiar. The following is an example.

1. Two weeks ago I had 6.7 tons of coal in my coal bin. The first week I burned 1.1 tons. The second week I burned only .8 tons.

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1. Woody, Clifford, Types of Arithmetic Needed in Certain, Types of Salesmanship, Elementary School Journal, Volume 22, page 505 - 520

2. Fourth Yearbook, Department of Superintendence, National Education Association, Chapter V Arithmetic page 177

How much coal have I left?

If written problems are developed out of the experiences and activities of the pupils, most of the problem difficulties would disappear. Scientific stodies on problem solving agree that most of the problem difficulties have been due to the fact that pupils did not have the experience necessary for understanding the situations presented in the textbook problems.[1]

The Compass Diagnostic Tests afford only a sampling of the useful facts, primary and related in the fundamental processes, and should, therefore, be classed as a survey test. Tests I to IV deal with the fundamental processes and may be used for general survey purposes. Tests V to XX are not in harmony with nor do they reinforce the right curricular principles.[2] They test useless and traditional material. As a whole the Compass

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1. Fourth Yearbook, Department of Superintendence National Education Association, Chapter V Arithmetic page 177.

2. Wilson, G.M., and Hoke, K.J., <u>How to Measure</u> The Macmillan Company, New York, 1929, page 279

Diagnostic Tests in Arithmetic are of doubtful value to the classroom teacher.

The Monroe Diagnostic Tests are composed of a series of twenty one tests designed to diagnose the particular difficulties of each individual child. These tests are printed in four parts. Tests I to XI of the Monroe Diagnostic Tests consist of examples on integers and are designed for gradesIV to VIII. Part III is on common fractions, and is designed for grades V to VIII; Tests XVIII to XXI (Part IV) consist of examples in multiplication and division of decimals. They are to be used in Grades VI to VIII. Each test is timed separately.

In Part 1, 75 of the 100 primary addition facts, 84 of the 300 upper decade facts are tested; 90 of the 100 multiplication facts, and 49 of the 449 short division facts are tested. In Part II, 25 of the 100 primary addition facts, 84 of the higher deacde facts; 54 of the 100 subtraction facts; and 64 of the 100 multiplication facts are tested. The Monroe Diagnostic Tests in Arithmetic do not meet the requirements of inventory tests. They do not afford

a complete inventroy of the useful facts, primary and related, in the fundamental processes. They afford a sampling of facts and therefore, may be classed as a survey test.

Test VII, Part II goes beyond social usage.[1] It consists of single column addition with thirteen addends.

Most of the fraction combinations are used only in textbooks of arithmetic, and not in real life situations, for example, 4/15 + 5/9; 7/9 - 1/6; 7/12 x 4/7; 4/7 ÷ 8/11. There is little or no need for the multiplication of a fraction by a fraction such as 3/4 x 5/6. The case for division of fractions is even less favorable.[2]

In business and social usage fractions are used mainly in connection with United States money, therefore, most of the work in decimals in the schools

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1. Wilson, G.M., What Arithmetic Shall We Teach? Houghton Mifflin Company, Boston, Massachusetts, 1926, page 124

2. Upton, Clifford B., Changing the Curriculum in Arithmetic, Teachers' College Record; 341 - 359, December 1926

should be confined to United States money, Otherwise the work is meaningless and involves mere manipulations.

By omitting Tests VII, Part II, which goes beyond social usage, Parts I and II of the Monroe Diagnostic Tests in Arithmetic may be used for general survey purposes. Parts III and IV are not in harmony with, nor do they reinforce the right curricular principles.[1] They test useless and traditional material in fractions and dedimals.[1]

The Buswell-John Diagnostic Chart aims to cover the four fundamental processes. The examples are arranged in the order of increasing difficulty, thus enabling the teacher to determine the specific difficulties of each pupil . The purpose is to give the child the greatest possible help in improving his work. This purpose is in line with the newer ideas of testing. The chief limitation of this diagnostic chart is the incomplete sampling of facts in each process. Only 69 of the 100 primary addition facts,

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1. Wilson, G.M., and Hoke, K.J., <u>How to Measure</u>, The Macmillan Company, New York, 1929, page 279

49 of the 300 upper decade facts, and 7 of the 80 higher decade facts are tested; 63 of the 100 primary subtraction facts, 77 of the 100 multiplication facts, and 52 of the 449 short division facts are tested. The results of the test would not give any information of the pupil's knowledge of the missing combinations thus making an incomplete inventroy.

The Wilson Inventory and Diagnostic Tests afford an almost complete inventory of useful facts, primary and related in the fundamental processes of arithmetic. The 100 primary addition facts are tested, 298 of the 300 upper decade facts, and 79 of the 80 higher decade facts are tested; the 100 subtraction facts, 99 of the 100 multiplication facts, and 447 of the 449 short division facts are tested. The Wilson Inventory and Diagnostic Tests afford a detailed analysis of process step difficulties. They were constructed on the basis of a careful analysis of the skills in each process. The fundamental purpose underlying inventory testing in arithmetic is to produce improvement in a pupil's

work by discovering and removing the incorrect habits of work which are the cause of his errors. The success of remedial teaching depends upon the accuracy and detail with which the specific skills are identified and isolated in an inventory test.

The Wilson Inventory and Diagnostic Tests are essentially inventory tests. They give an almost complete inventory of useful facts, primary and related in the fundamental processes. They afford a detailed analysis of process step difficulties thus enabling the teacher to make a careful analysis of the pupil's work, and on that basis to do whatever reteaching is necessary and to prescribe the necessary remedial work. The Wilson Inventory and Diagnostic Tests conform to social usage. They satisfy the requirements of the criteria of a standardized test. They are to be highly recommended for use in the classroom. They will be most helpful to teachers in increasing the effectiveness of their instruction.

Dr. W.J. Osborn, author of the Wisconsin Inventory

Tests in Arithmetic, was one of the first to recognize that each useful fact, primary and related, must be mastered before 100% accuracy can be assured.

The Wisconsin Inventory Tests endeavor to test each useful fact in the fundamental processes. They test each of the 100 primary addition facts, 227 of the 300 upper decade facts, 79 of the 80 higher decade facts; 98 of the 100 primary subtraction facts, the 100 multiplication facts and 162 of the 449 short division facts. In addition, subtraction, and multiplication, the Wisconsin Inventory Tests give an almost complete inventory of the useful facts, primary and related, but in short division only about 37% of the useful facts are tested. The work in fractions and decimals is simple. The Wisconsin Inventory Tests in the fundamental processes may be profitably used.

The Wilson Inventory and Diagnostic Tests, and the Wisconsin Inventory Tests in Arithmetic, afford the most satisfactory inventory of useful facts, primary and related, of the tests analyzed.

The Wilson Inventory and Diagnostic Tests are

slightly superior in testing more adequately the number of useful facts primary and related in the fundamental processes and much superior in the detailed analysis of process step difficulties.

Care is necessary in choosing a standardized test. Before using a test it should be carefully examined; first as to basic curricular values and effects on methods of teaching; second as to possible value of details in inventorying pupil needs and improving teaching. Much needs to be done.

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